Engineering for Particle Accelerators - Magnets

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Education in Beam Physics and Accelerator Technology

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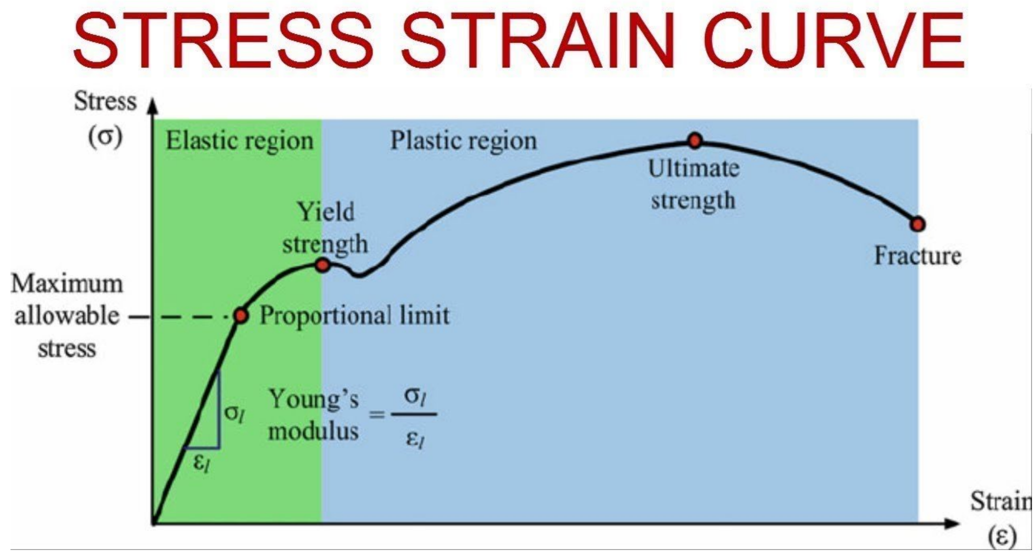
Outlines

- Chapter 1: Basic Principle
- Chapter 2: Magnets in Accelerator
- Chapter 3: Magnetic Field Equations
- Chapter 4: Dipole Magnet Design and Manufacturing
- Chapter 5: Quadrupole Magnet Design and Manufacturing
- Chapter 6: Solenoid Design and Manufacturing
- Chapter 7: Superconducting Magnet Design and Manufacturing

Chapter 1 Basic Principles

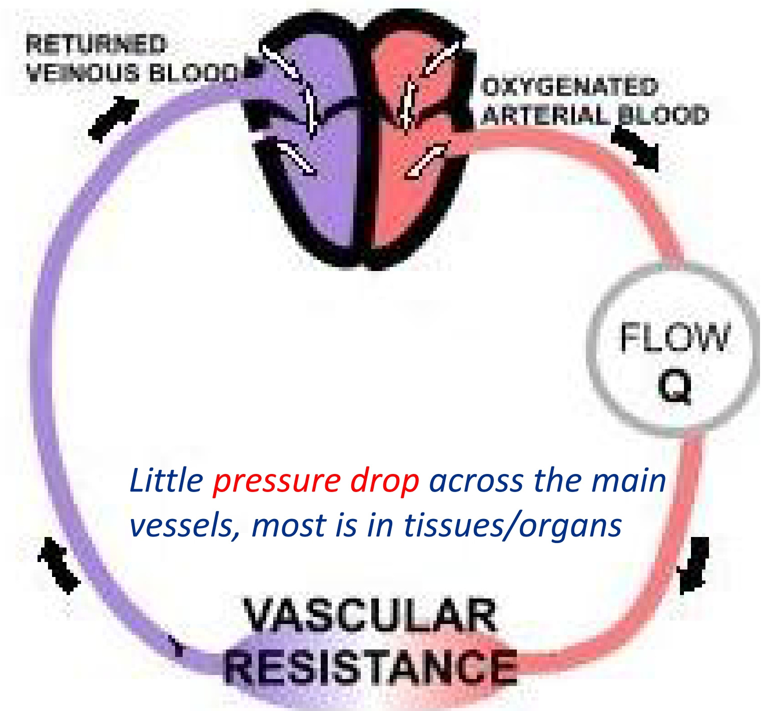
Basic principles: Mechanics

brittle fracture deformation fracture



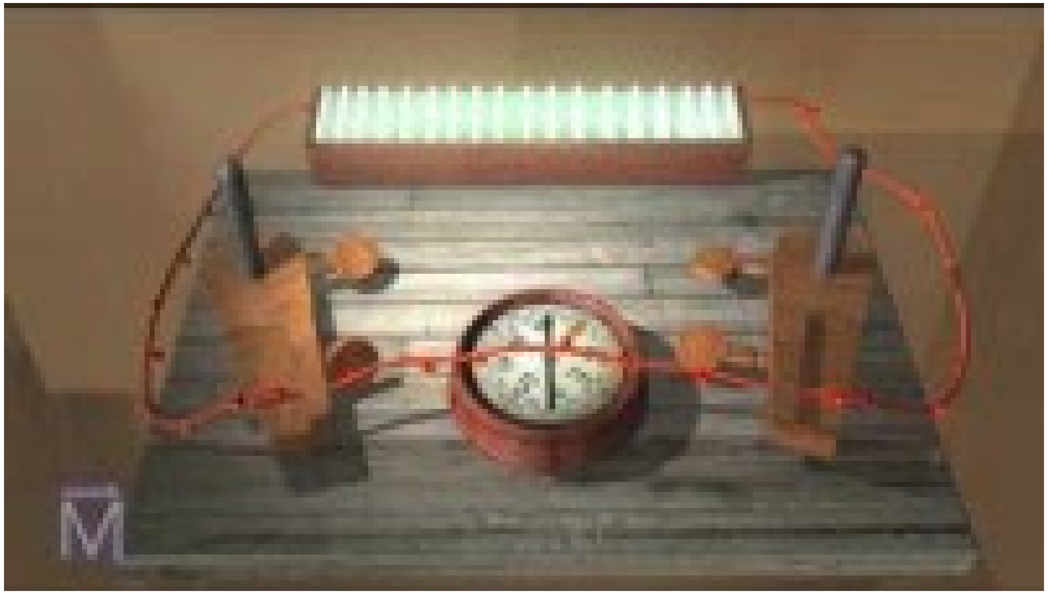
- To elongate the rod until its break, you need a force.
- $\sigma = F/A$, $\varepsilon = \Delta L/L$, where σ is stress (Pascal/Psi), F is force (N/Lbf), A is the cross-section area (m^2/in^2), ε is strain and L is length.
- Young's modulus (or Elastic modulus) $E = \sigma / \varepsilon$.
- With known E and measured ε, σ can be calculated.

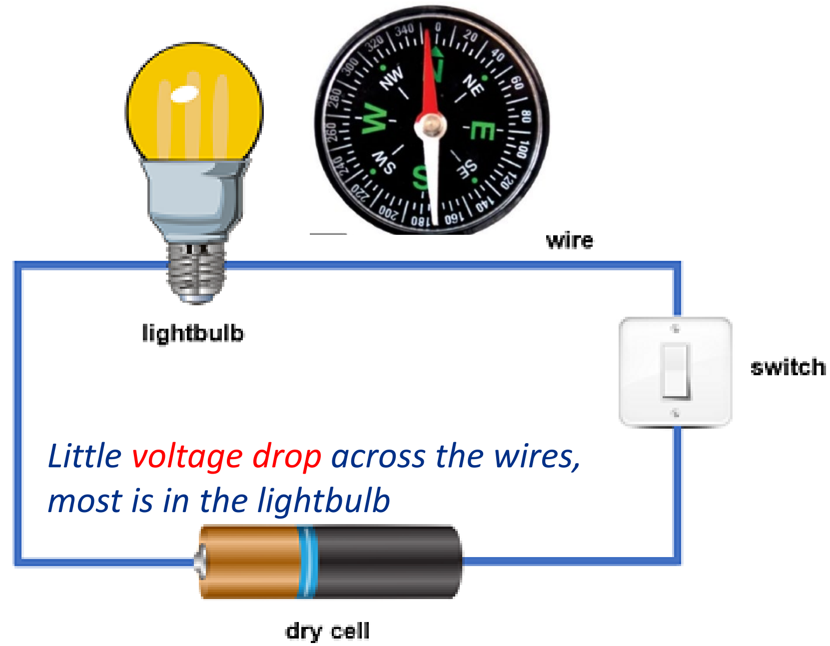
Basic principles: Hydraulic circuit



- To make fluid circulating, you need a pump for a pressure, creating the flow.
- $Q = \upsilon \cdot A$, $\Delta P = R \cdot Q$ where Q is flow rate (m^3/s or ft^3/s), υ is speed (m/s or ft/s), A is the cross-section area (m^2/ft^2), P is pressure (Pascal/Psi), and R is resistance or viscosity for fluid ($N \cdot s/m^2$ or lbf·s/ft2).

Basic principles: Electric circuit and Magnetism

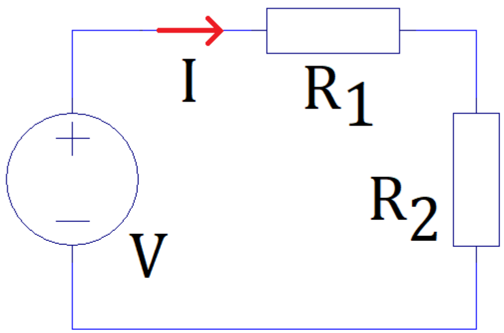




- To produce electric current, you need a generator for a voltage, creating the current flow.
- $I = C_{density} \cdot A$, $\Delta V = R \cdot I$ where I is current, $C_{density}$ is current density, A is the cross-section area, V is voltage and R is resistance.
- Put a compass needle by the circuit. When switch on the circuit, the compass needle is deflected by the electric current - Oersted’s law.

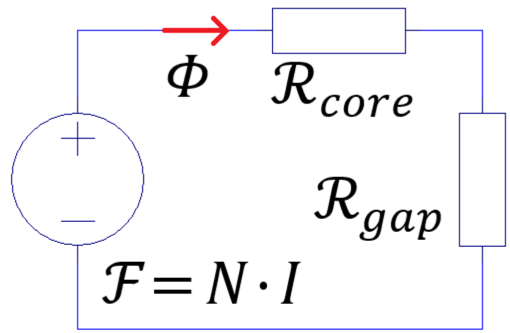
Basic principles: Magnetic circuit

electric

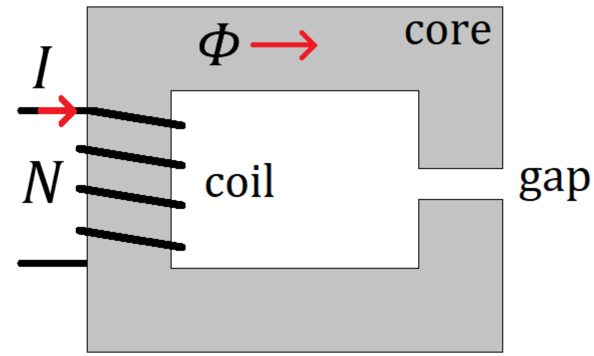


$$I = \frac{V}{R_1 + R_2}$$

magnetic



$$\Phi = \frac{\mathcal{F}}{\mathcal{R}_{core} + \mathcal{R}_{gap}}$$



C-shape magnet

- In a magnetic circuit,
  - the source is represented by the magnetomotive force $\mathscr{F}$ (A·turns)
  - the analogue of the electric current is the magnetic flux Φ (Wb)
  - the analogue of electrical resistance is the magnetic reluctance $\mathscr{R}=\frac{L}{\mu A}$, L is the magnetic circuit length and A is the cross-section area of the circuit.

Units in SI for a Magnet

Variable	Unit
F (force)	Newtons (N)
q (electric charge)	Coulombs (C)
B (magnetic field density)	Tesla (T)
I (current)	Amperes (A)
E (energy)	Joules (J)

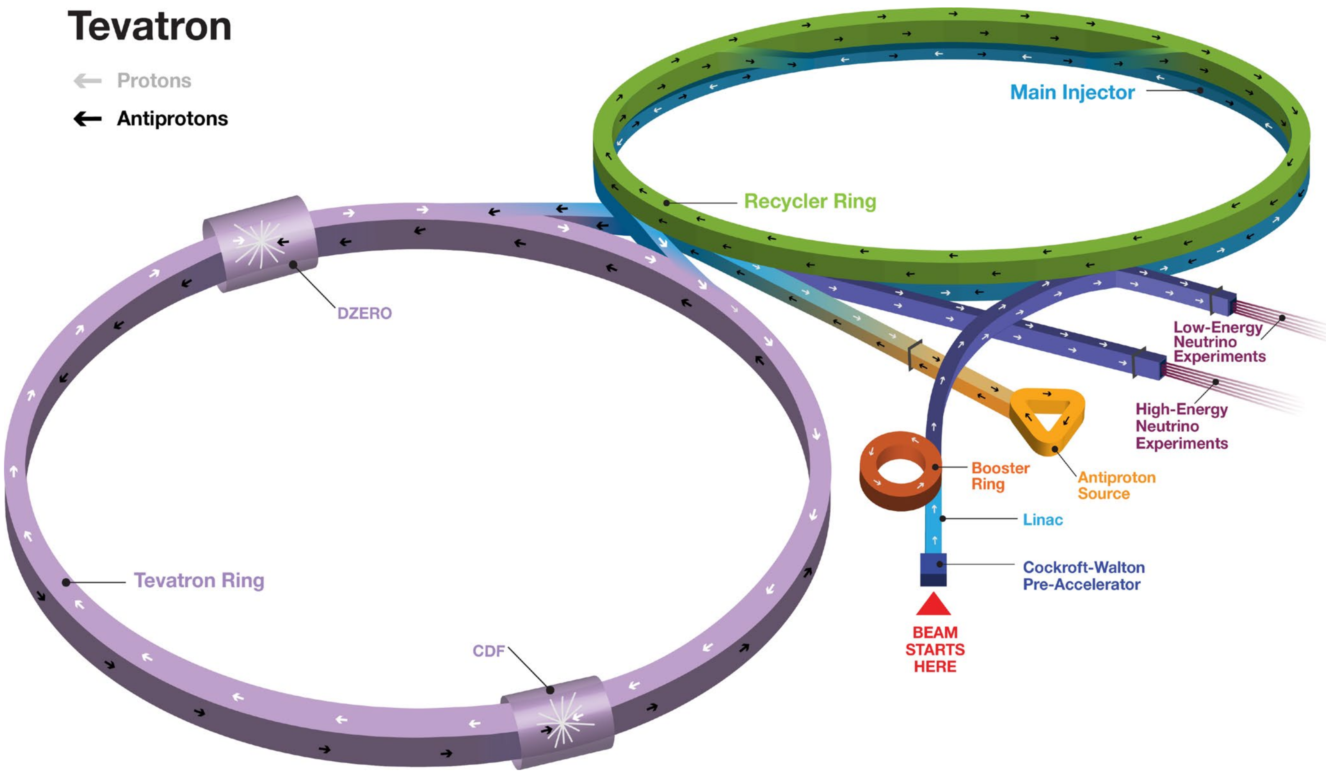
- 1 T = 10,000 Gauss (G)
- $\mu_0 = 4 \pi \times 10^{-7}$ (T·m/A), free space permeability
- Charge of 1 electron ~1.6×10^{-19} C, so for particle beams 1 eV = 1.6×10^{-19} J

Magnitude of Magnetic Fields

Value	Item
0.1-1.0 x 10^{-12} T	Human brain
24 x 10^{-6} T	Magnetic tape near tape head
31-58 x 10^{-6} T	**Earth's magnetic field at 0° latitude (on the equator)**
0.1 x 10^{-3} T	Exposure limit for cardiac pacemakers
1 x 10^{-3} T	**A typical refrigerator magnet**
0.25 - 0.4 T	A sunspot
1.2 T	An earth magnet
1.5 – 3.0 T	**Medical MRI system in practice**
9.4 T	Modern high resolution research MRI system
11.7 T	NMR spectrometer
16 T	To levitate a frog
45 T	The strongest continuous manmade magnetic field in a US laboratory
100 T	Strongest (pulsed) magnetic field (non-destructively) in a US laboratory
1.2 x 10^{3} T	Strongest (pulsed) magnetic field (destructively) in a Japan laboratory
2.8 x 10^{3} T	Strongest (pulsed) magnetic field (explosive) in a Russia laboratory
1-100 x 10^{6} T	Neutron star's magnetic field
0.1-100 x 10^{9} T	Magnetar's magnetic field

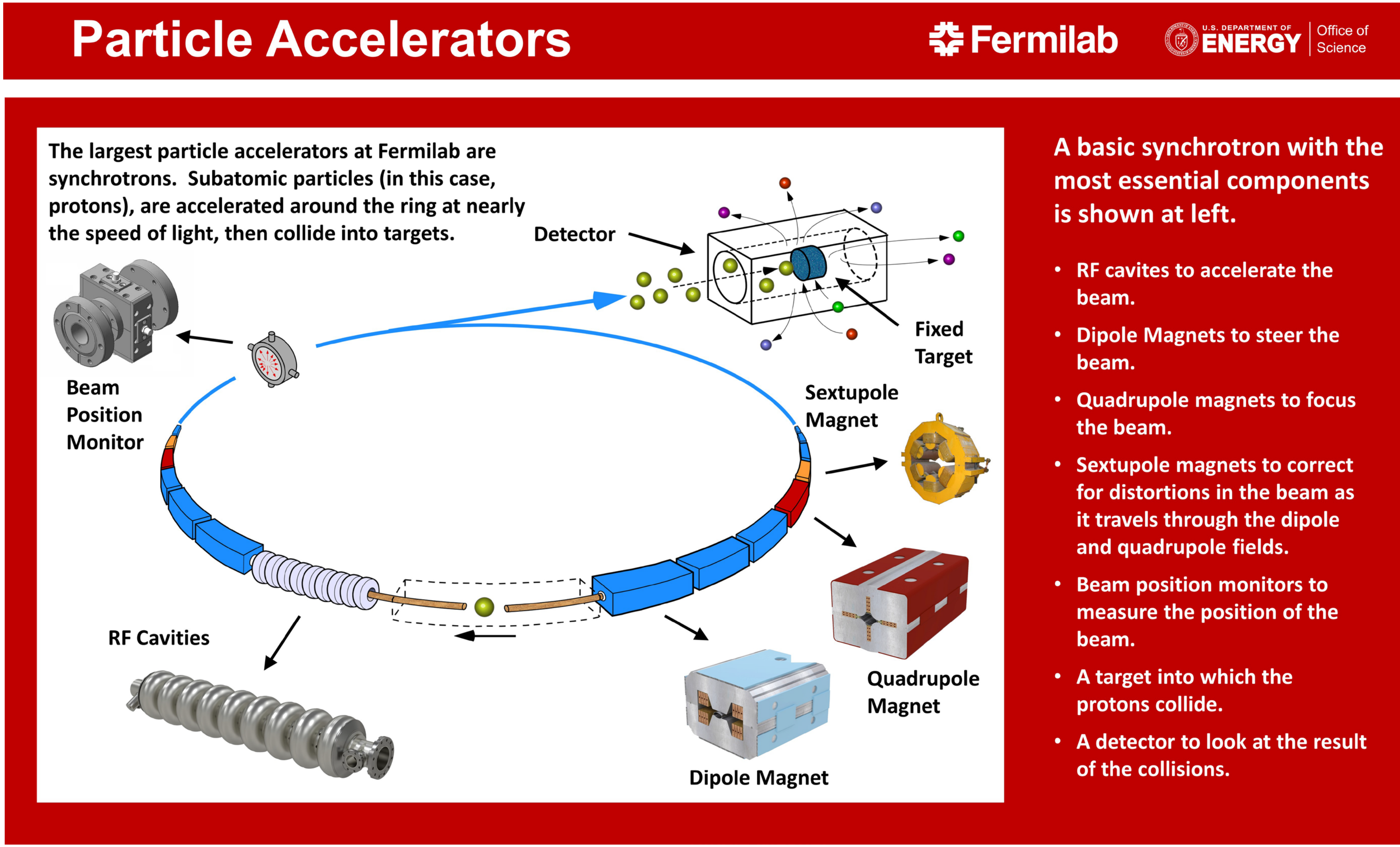
Chapter 2 Magnets in Accelerator

The Tevatron at Fermilab



- The Tevatron (6.28km/3.90mi tunnel), located at Fermilab, was the world's highest energy particle accelerator from 1983 until 2009, when it was exceeded by the LHC (27kg/17mi tunnel) at CERN, Switzerland.
- Protons and anti-protons circulated in opposite directions in the same beam tube and collided at the CDF and DZero detectors with energies of nearly 2 trillion electron volts (2 TeV). The Tevatron was shut off in 2011, allowing the remaining components of the Fermilab campus to be reconfigured for new experiments.

Fermilab Main Injector (MI)

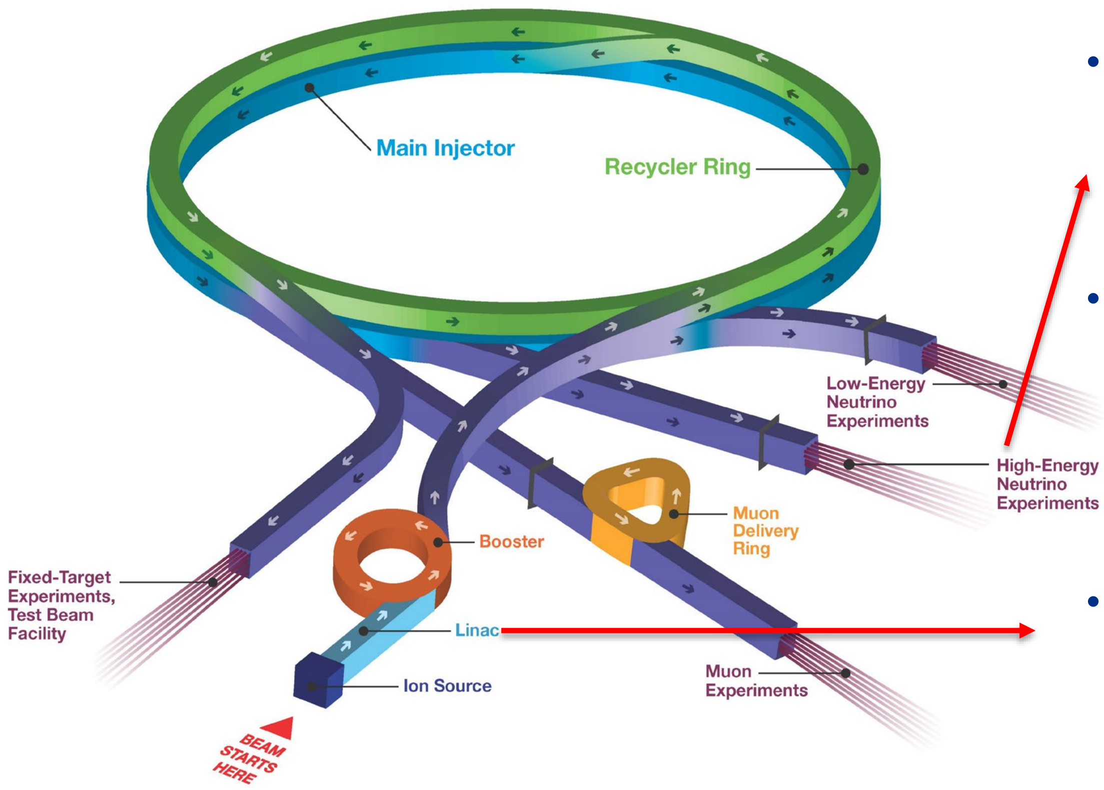


Magnets in MI Particle Accelerator

- The two-mile-long tunnel at Fermilab houses the Main Injector particle accelerator and the Recycler particle storage ring.
- Permanent magnets (green) steer protons around the storage ring before they are transferred to the accelerator (blue and red) and accelerated close to the speed of light.
- The protons are then sent to various physics experiments as well as the Fermilab Test Beam Facility.

Fermilab's Future

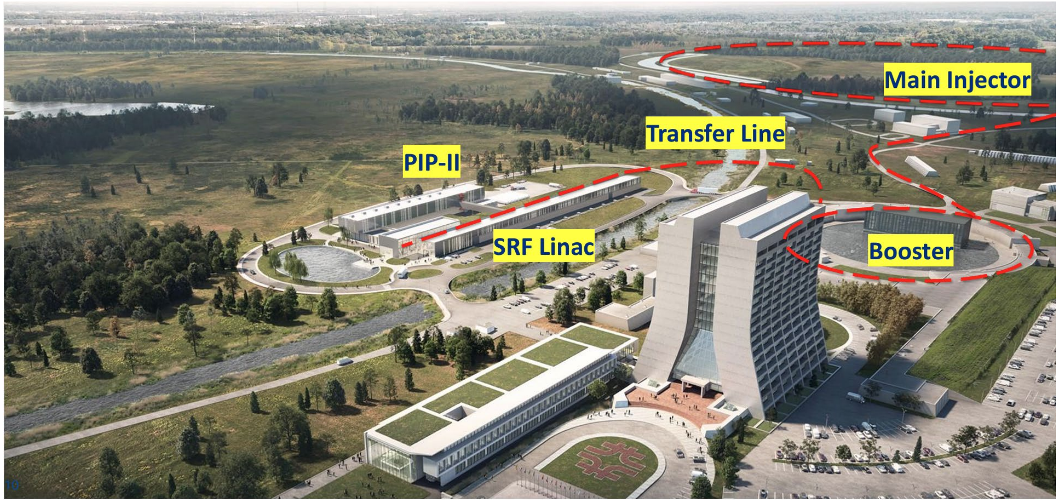
Fermilab is currently being reconfigured to become the world's premier "high intensity" accelerator. Extremely intense beams of particles such as muons and neutrinos are sent to collide into experimental apparatuses (targets) located in detector halls on the Fermilab site as well as in such faraway locations.

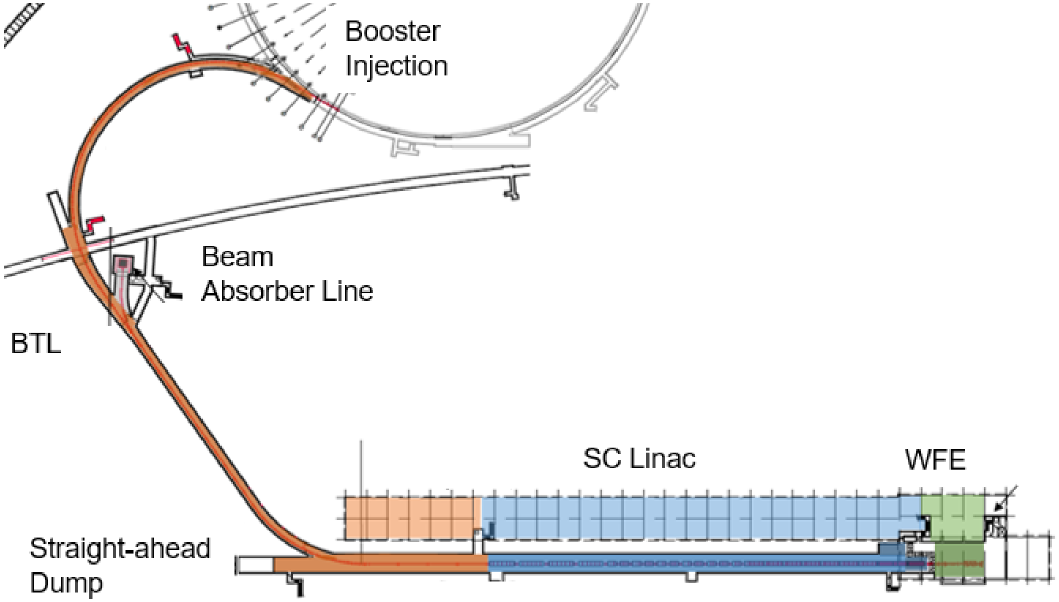


- PIP II enables the Long-Baseline Neutrino Facility (LBNF) to produce the world's most intensive neutrino beam.
- The beam will be used for the Deep Underground Neutrino Experiment (DUNE).
- The Proton Improvement Plan II (PIP II): SC Linac upgrade will accelerate protons up to 800 MeV over 215 m length

Fermilab PIP II Overview

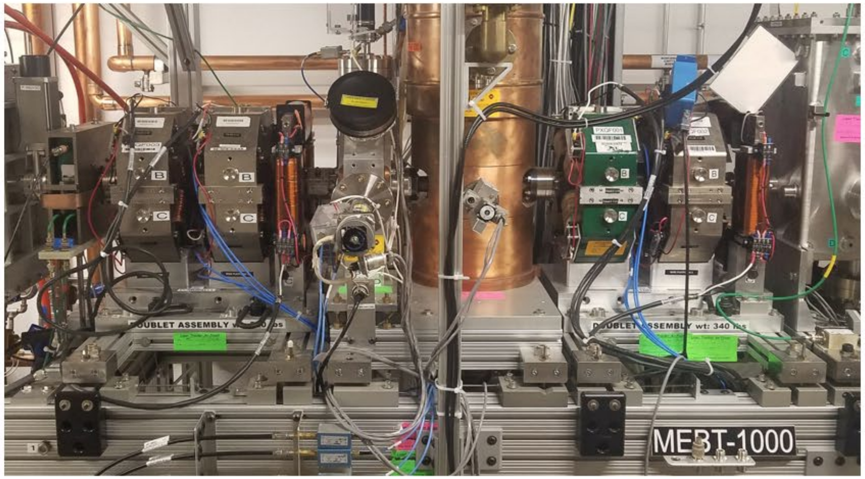
Magnets in PIP II

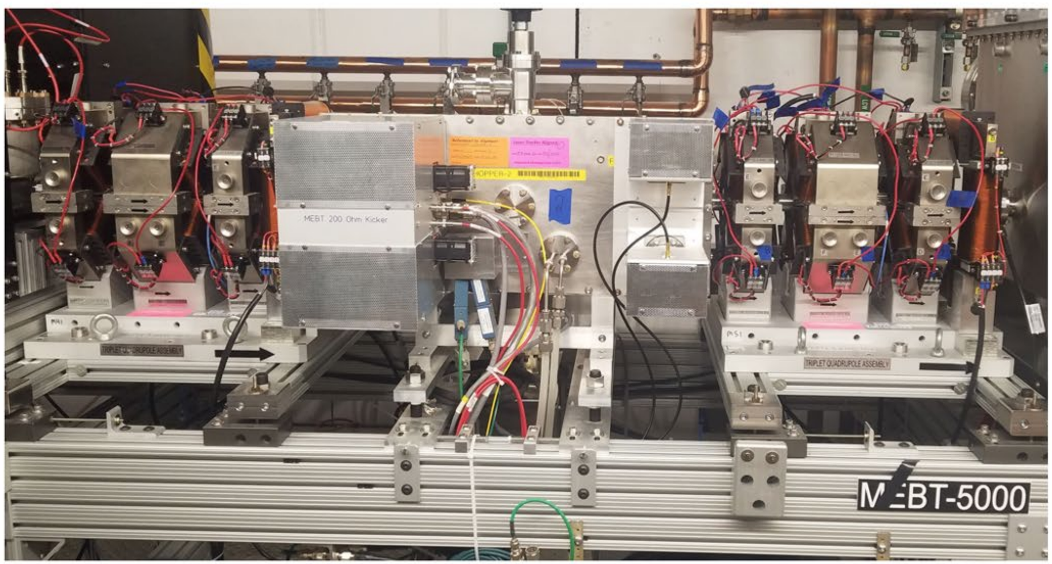


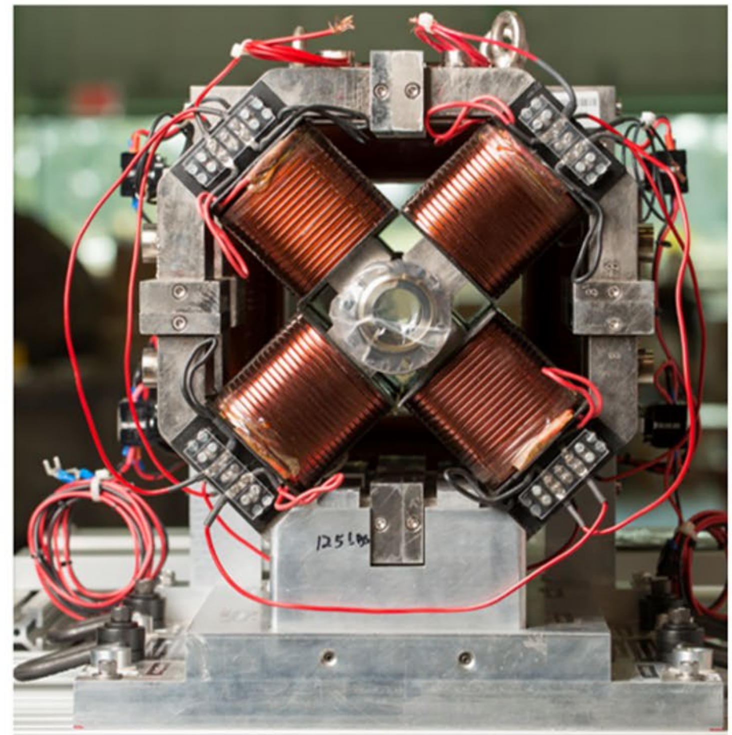


Magnet Type	Quantity
MEBT Long Quads	13
MEBT Short Quads	18
MEBT Correctors	11
HWR Solenoids	8
HWR Correctors	16
SSR1 and SSR2 Solenoids	29
SSR1 and SSR2 Correctors	116
650 MHz Warm Quads	40
650 MHz Warm Correctors	36
BTL Regular Dipoles	37
Vertical Booster Injection Dipoles	2
Regular BTL Quads	49
Large Aperture BTL Quads	2
EOL BTL Quads	6
BTL Correctors	56
3-way Septum	1
Fast Switch Magnet	1
Booster Injection C-magnet	1

PIP II WFE: Quadrupole Doublet and Triplet

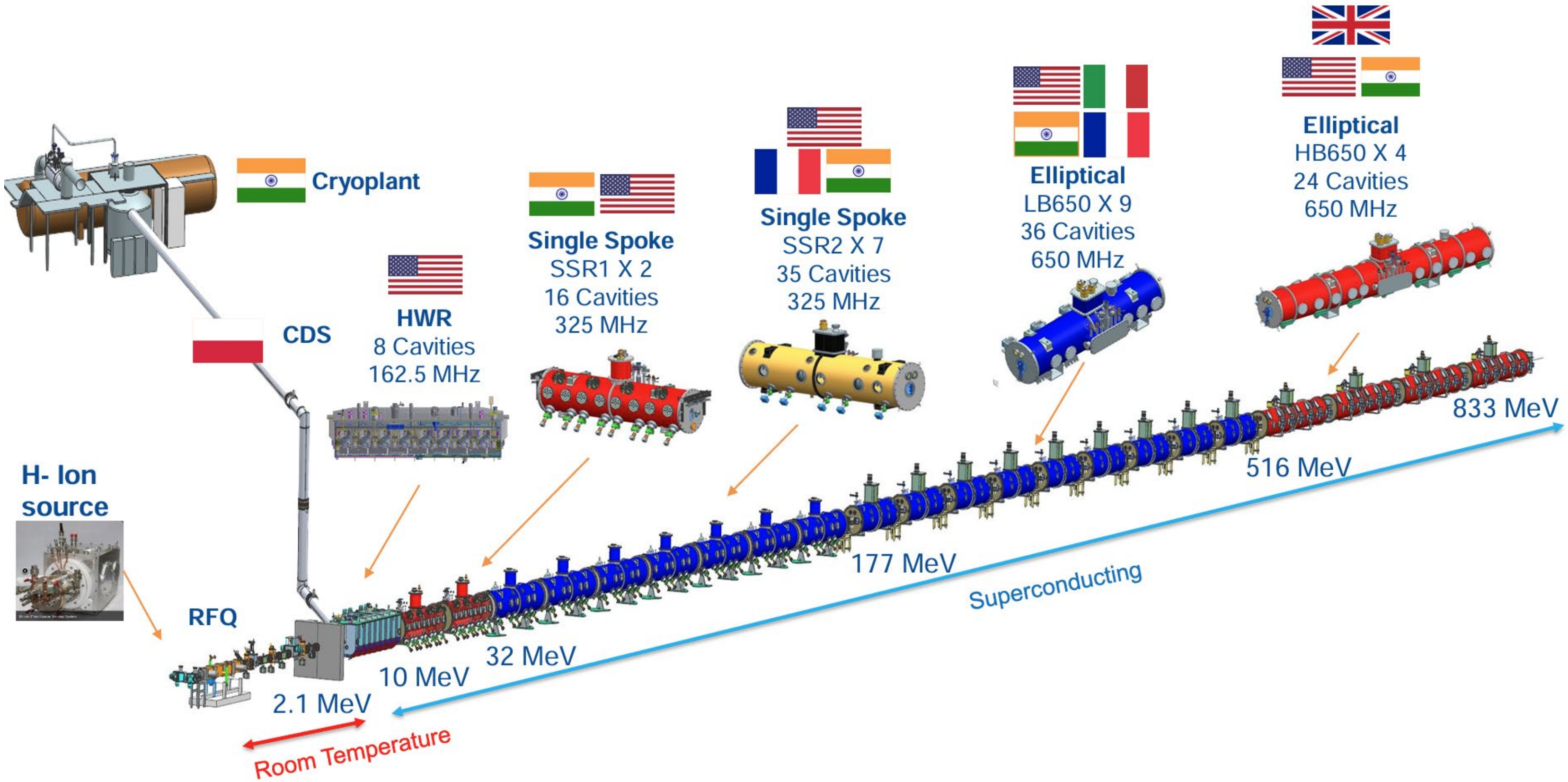






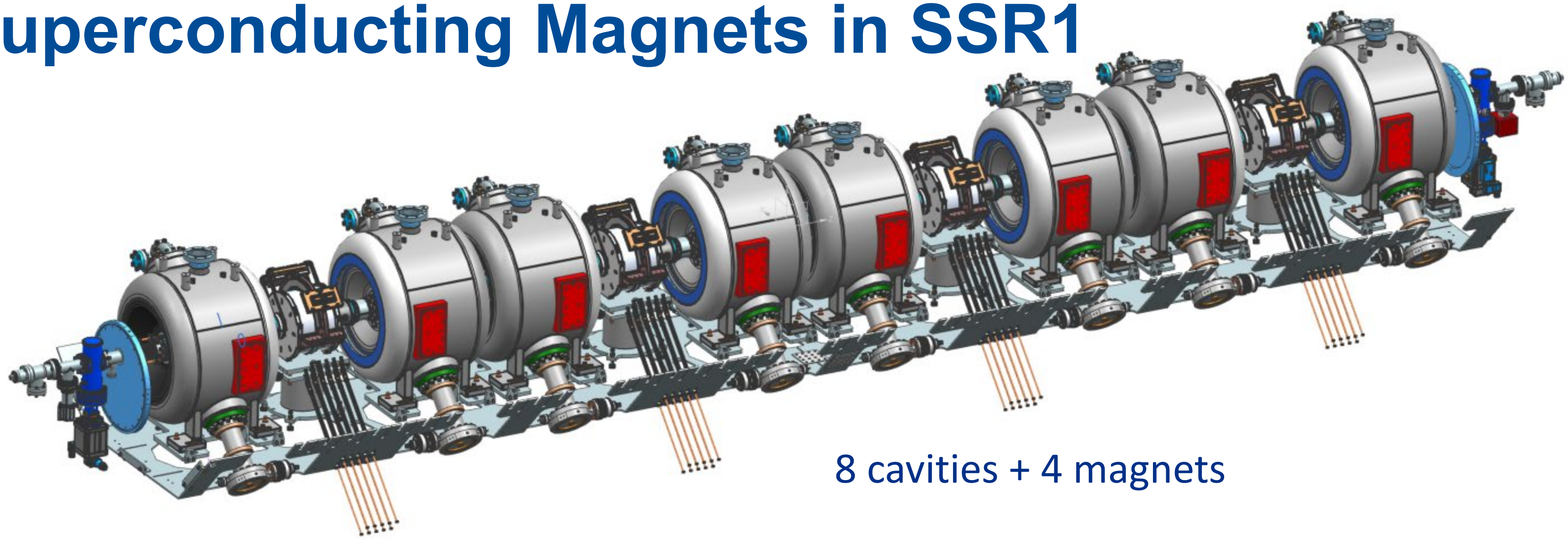
MEBT (Medium Energy Beam Transport) Quadrupole Magnets in PIP_II Linac

PIP II Linac



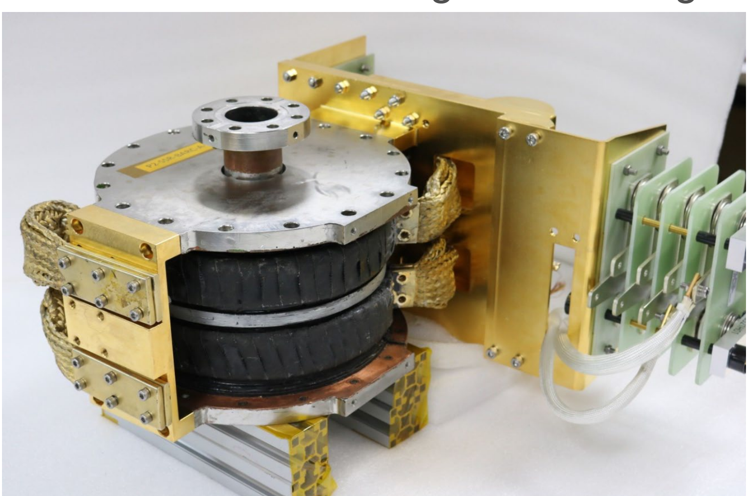
Section	CM Qty	Cav/Mag per CM	Energy (MeV)
HWR	1	8 / 8	2.1 – 10
SSR1	2	8 / 4	10 – 32
SSR2	7	5 / 3	32 – 177
LB	9	4 / 0 (2*)	177 – 516
HB	4	6 / 0 (2*)	516 – 833

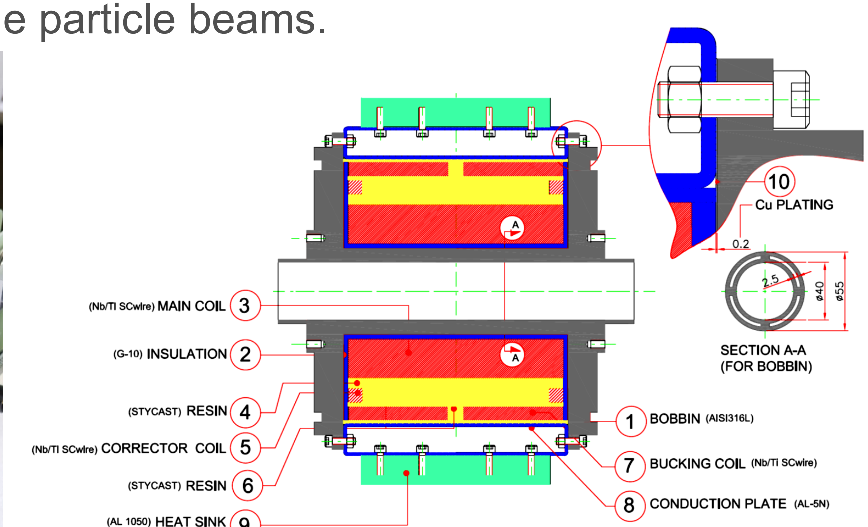
Superconducting Magnets in SSR1



8 cavities + 4 magnets

- High energy accelerators and beam transport lines for accelerators require higher fields not achievable with iron dominated magnets and must rely on superconducting technology.
- The solenoids in the string are focusing the particle beams.





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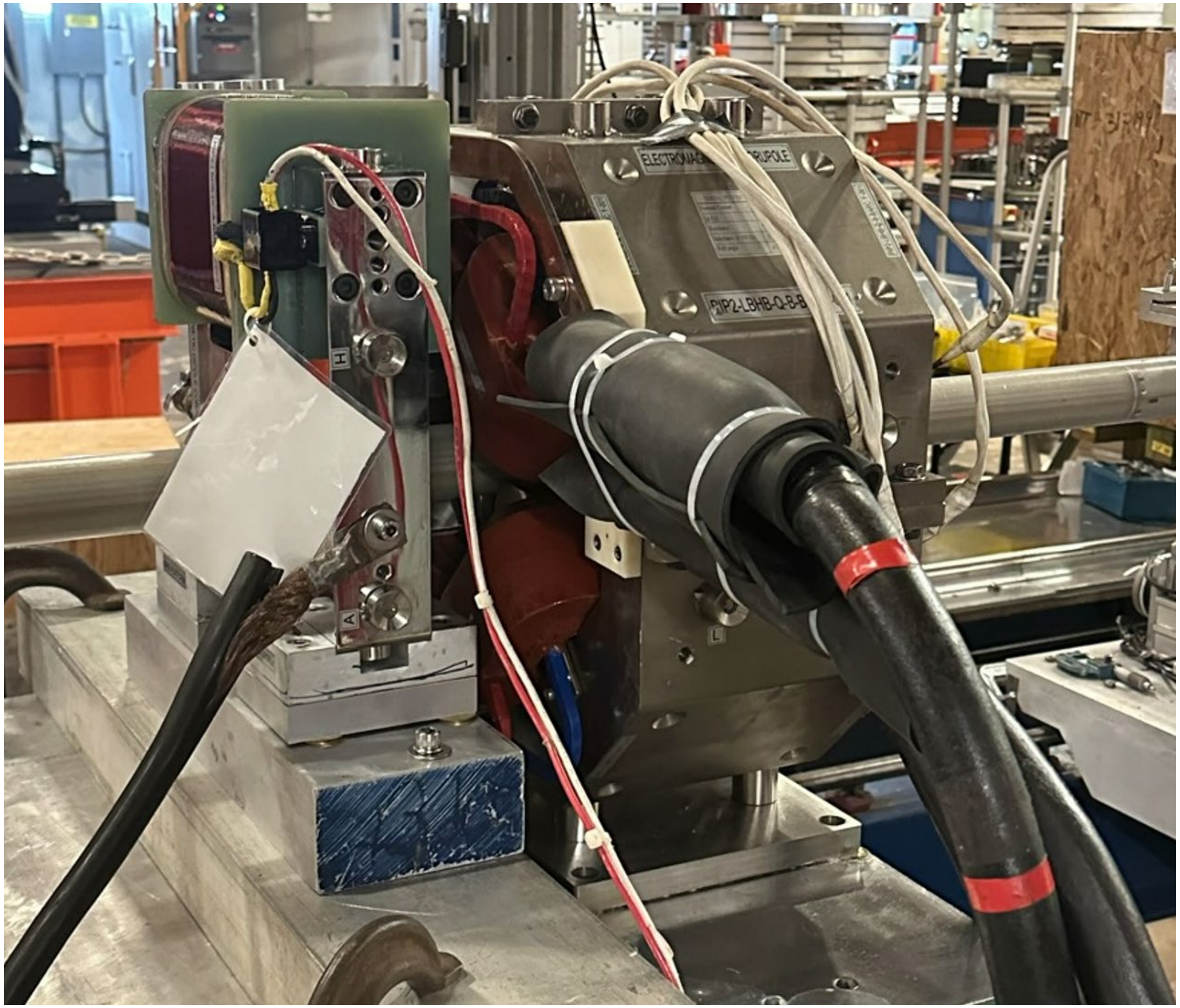
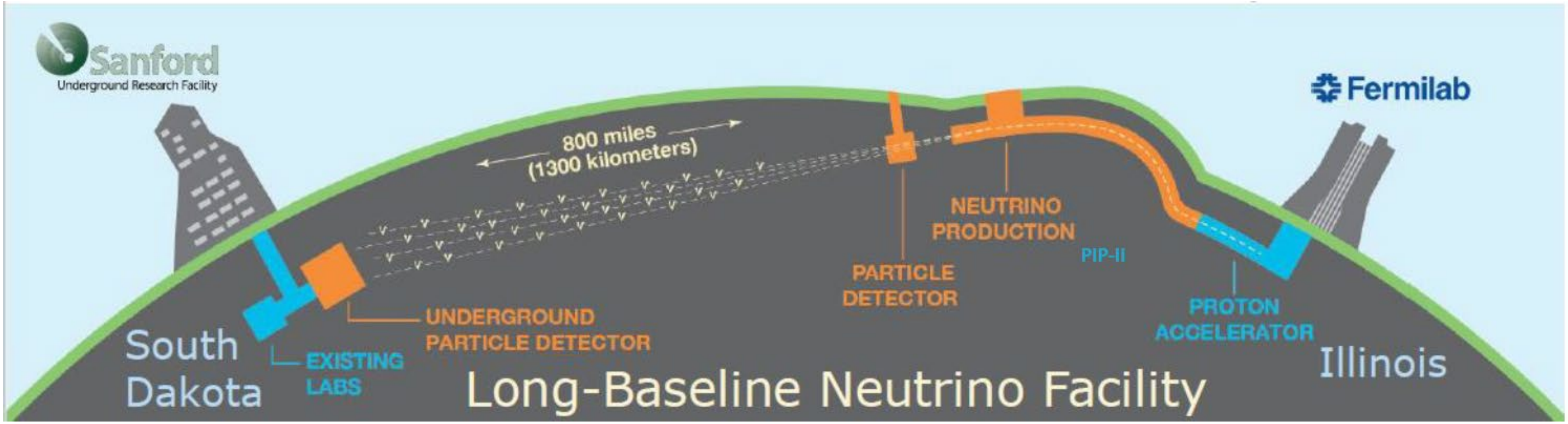
LBHB Magnets in PIP II



LBHB Doublet Magnets: Quadruple and Dipole Corrector

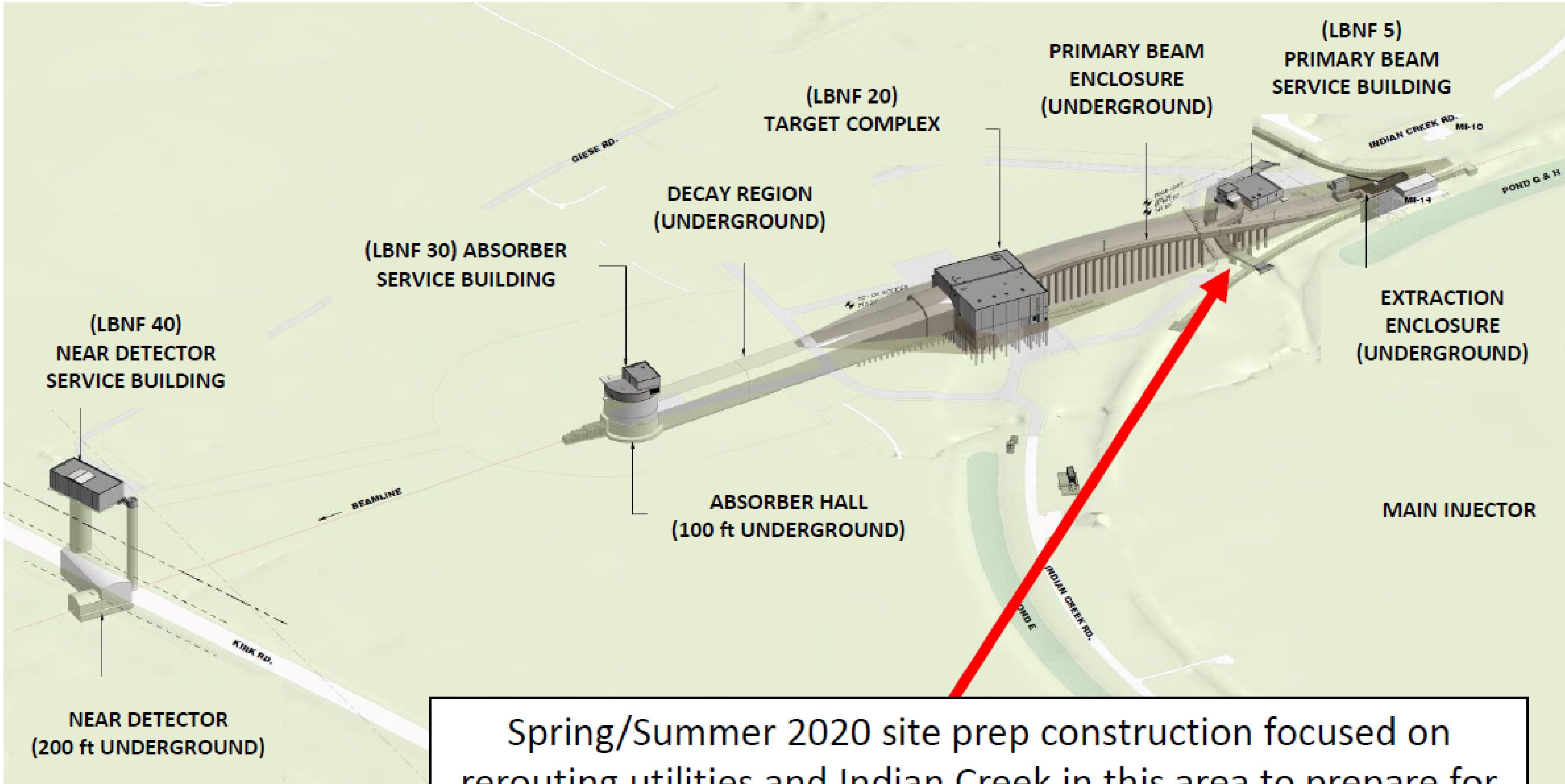
LBNF/DUNE Overview

LBNF/DUNE Overview_Cont.



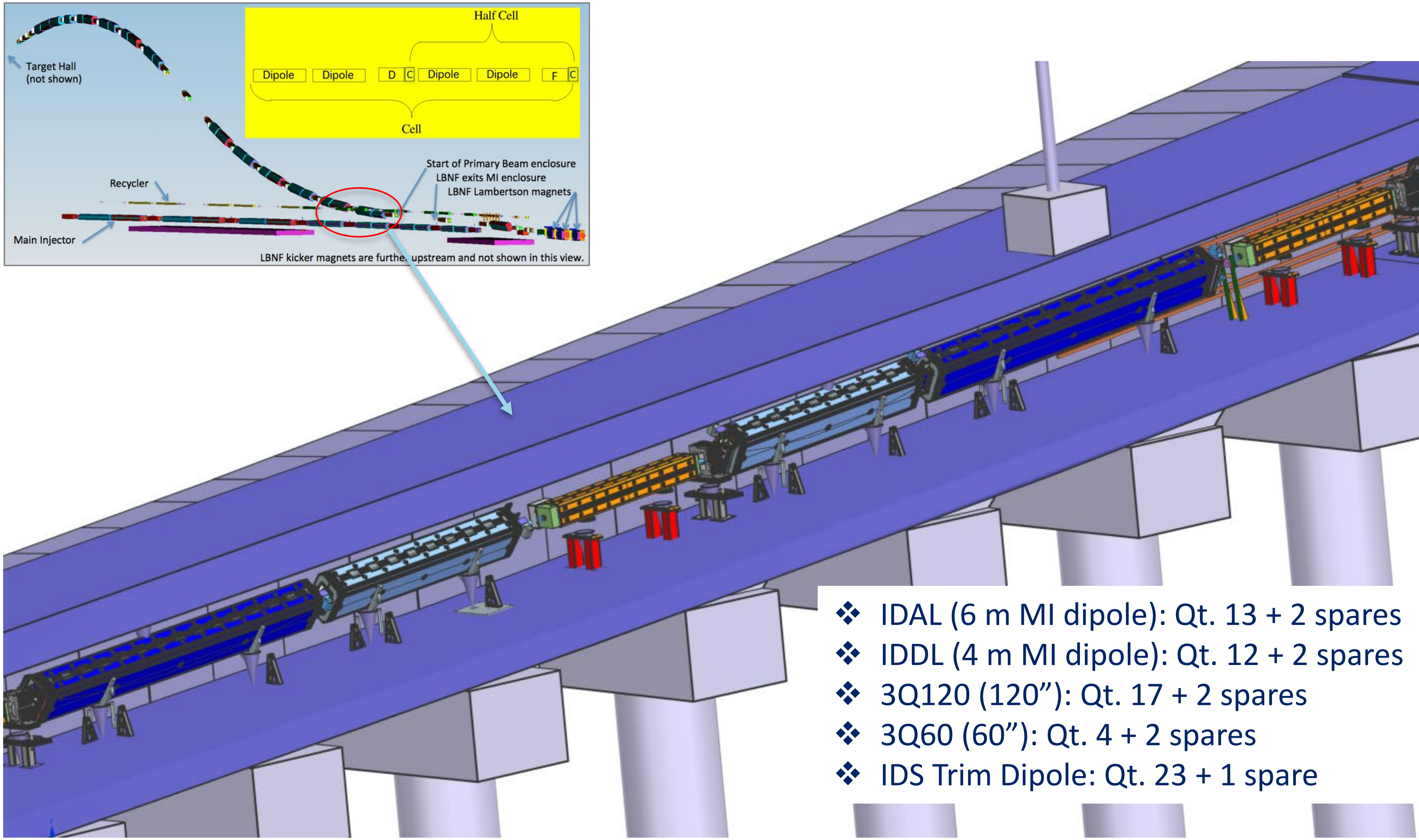
- Long Baseline Neutrino Facility (LBNF) – Platform to support the Deep Underground Neutrino Experiment (DUNE), provides:
  - ✓ Facilities at Fermilab, the “Near Site”, to produce world’s most intense neutrino beam and support the DUNE near detector
  - ✓ Facilities at the surface and 1.5 km underground at the Sanford Underground Research Facility (SURF), the “Far Site”, to support the DUNE far detector modules

LBNF Near Site Conventional Facilities



Spring/Summer 2020 site prep construction focused on rerouting utilities and Indian Creek in this area to prepare for the main LBNF construction planned to begin Fall 2023

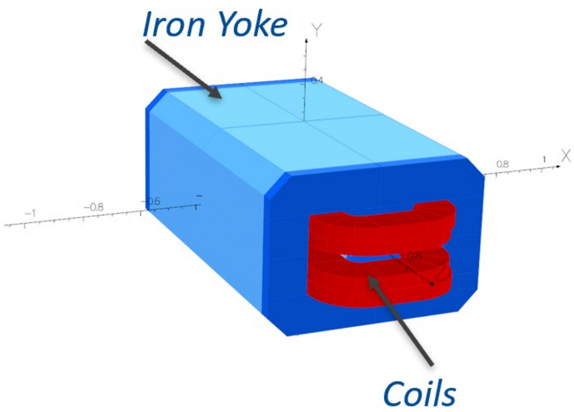
LBNF Primary Beamline Magnets Layout



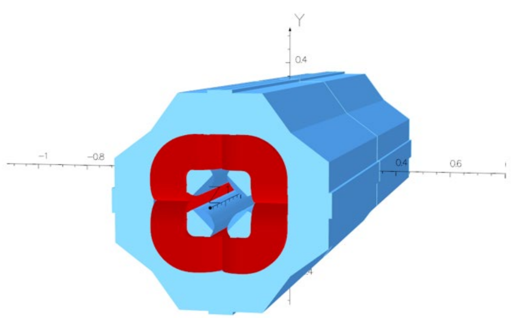
- IDAL (6 m MI dipole): Qt. 13 + 2 spares
- IDDL (4 m MI dipole): Qt. 12 + 2 spares
- 3Q120 (120"): Qt. 17 + 2 spares
- 3Q60 (60"): Qt. 4 + 2 spares
- IDS Trim Dipole: Qt. 23 + 1 spare

Conventional Magnets

- The magnet excitation is provided by current carrying coils.
- The field is shaped by iron poles.



FNAL Main Injector Dipole



FNAL MI WQB quadrupole

- ❖ Conventional magnets still “a working horse” for many accelerator magnet systems having fields below 1.8 T.
- ❖ They also could not be replaced by superconducting magnets for fast pulsed fields, and in very high radiation areas.
- ❖ Some accelerators have hundreds, or even thousands of the dipole and quadrupole magnets connected in series. In this case, cost optimization is needed, including the cost of fabrication and the cost of used electricity to power the magnets.
- ❖ For most water-cooled magnets, the optimal current density in copper coils is around 4 A/mm^2 (max. is 10 A/mm^2). Iron yoke made from solid, or laminated steel.

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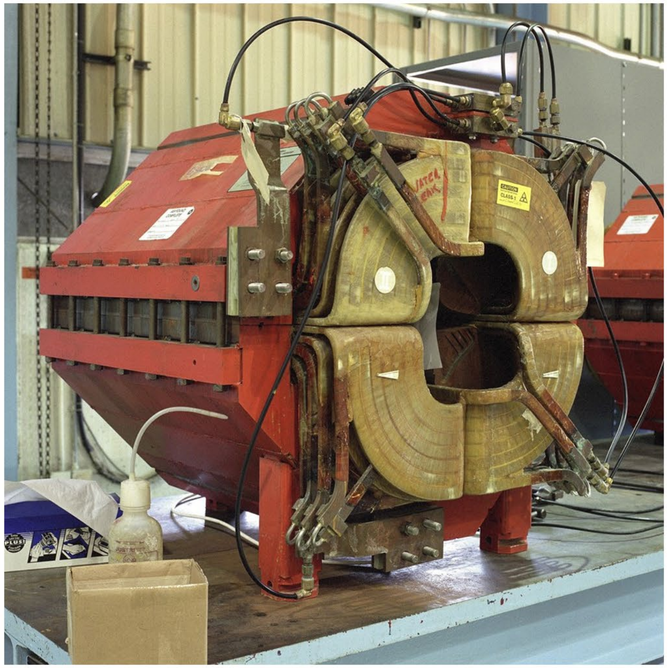
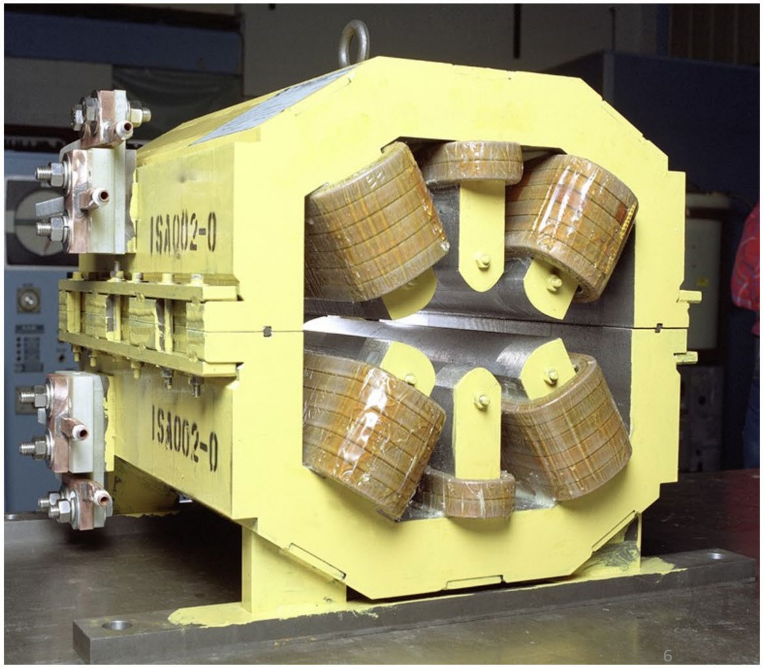
Conventional Magnets

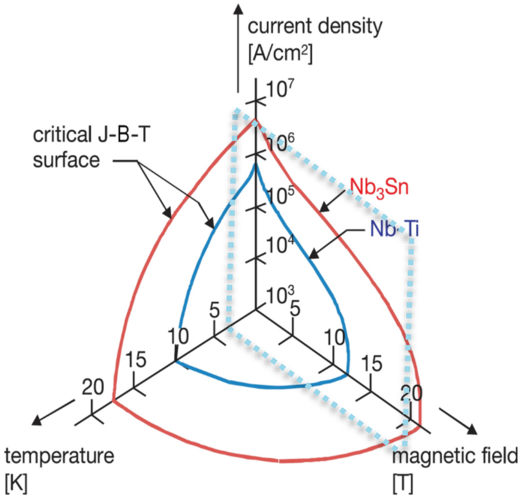




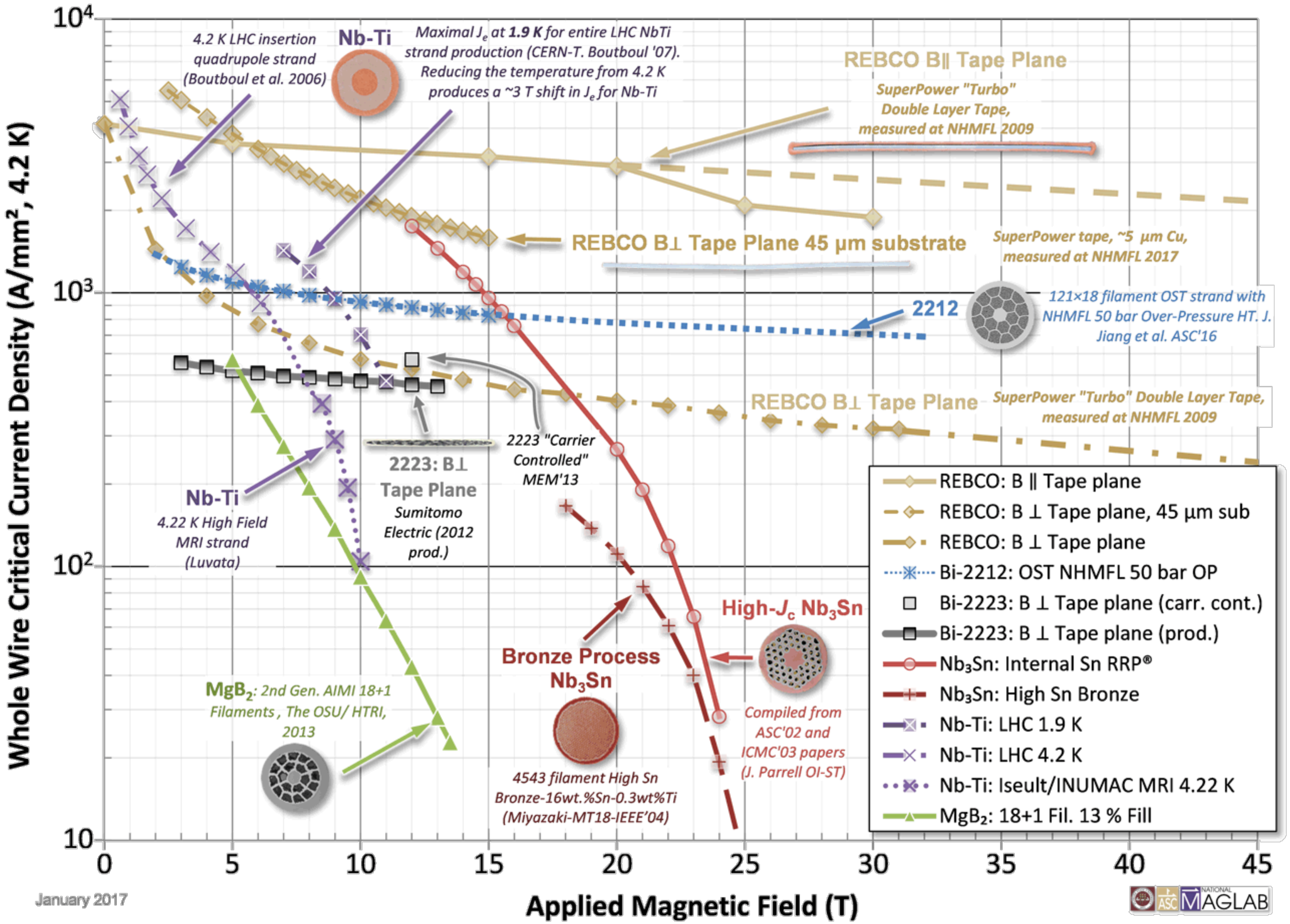
Superconductivity

- The metal becomes perfect conductors of electricity (zero resistance) when cooled to a critical temperature T.
- This superconductor, in addition to the critical temperature, also has a critical field B, above which it reverts to a normal resistive state.
- To fully prescribe the superconducting properties, the concept of a critical current density J is introduced.

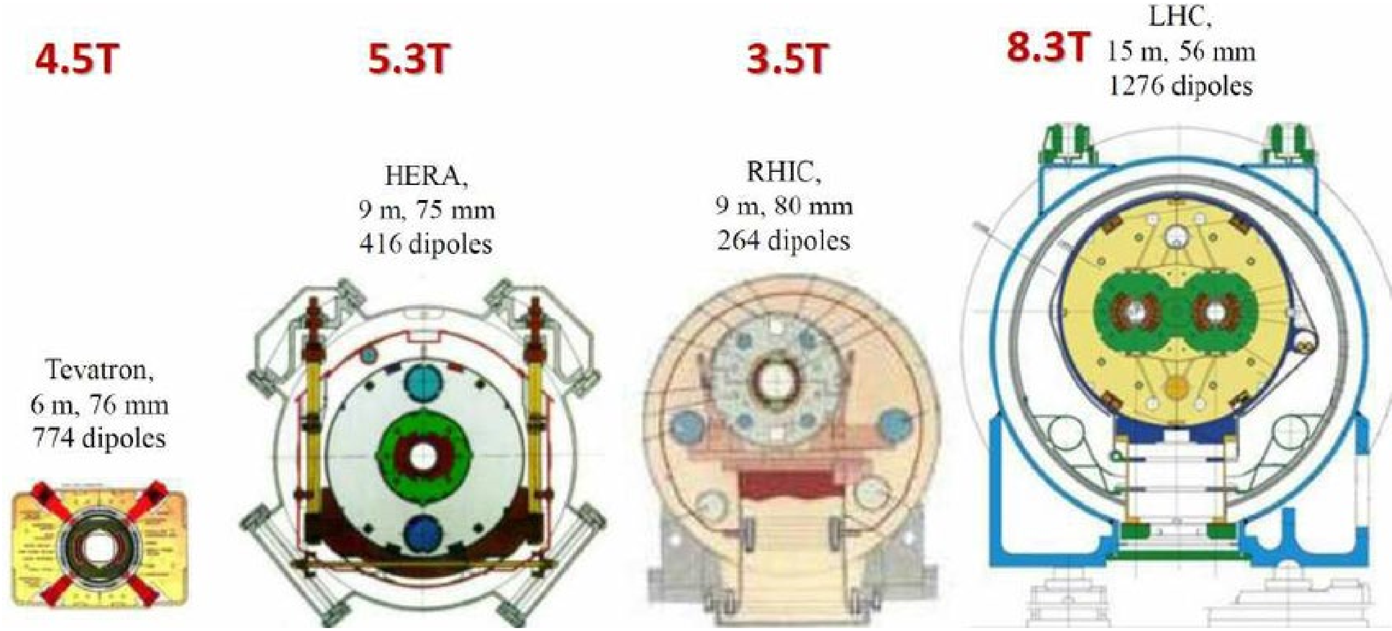
1. Superconductivity prevails everywhere below this surface with normal resistivity every where above it.
2. An increase in any one of the properties invariably produces a decrease in the other two.
3. The usual operating temperature for low-temp. superconducting magnets is 4.2 K.



Superconductors at 4.2 K

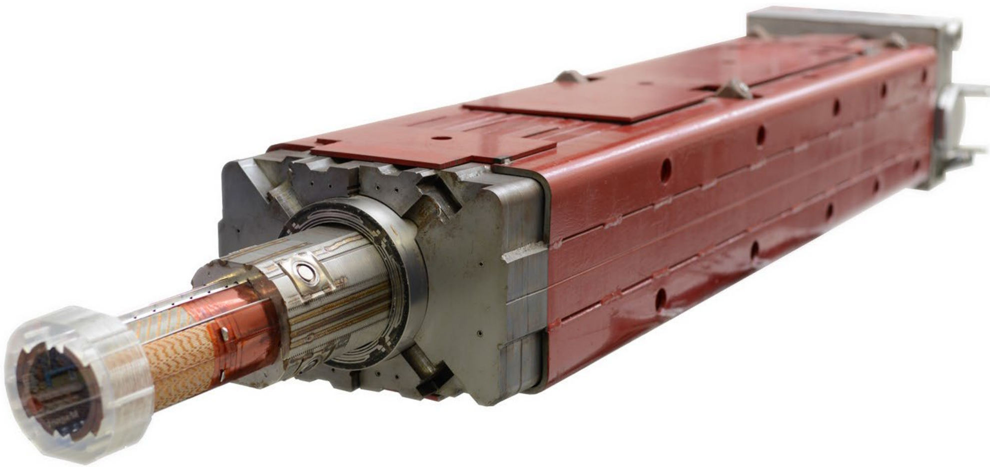


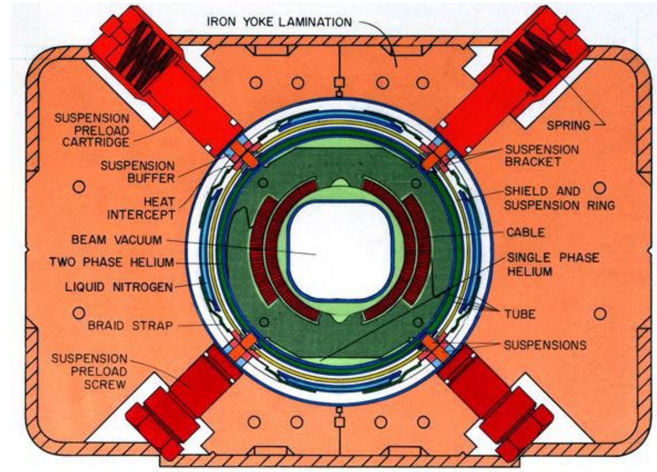
Superconducting Magnets



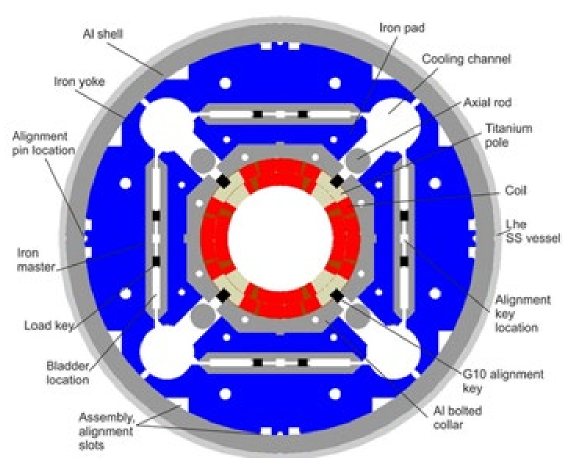
- Due to iron saturation, the conventional magnets cannot reach higher magnetic field (>1.8T). To meet the needs of the powerful accelerator, superconducting magnets were fabricated using NbTi Superconductor in the recent decades.
- Nb_3Sn magnets have a good progress in recent years for LHC upgrade.
- High-temperature superconductor (HTS) is in an R&D phase for accelerator magnets with two main directions: low field with high temperature up to 77K, and very high field with low temperature of LHe at 4.5K. Hybrid magnets with HTS and NbTi/Nb_3Sn are in the mainstream.

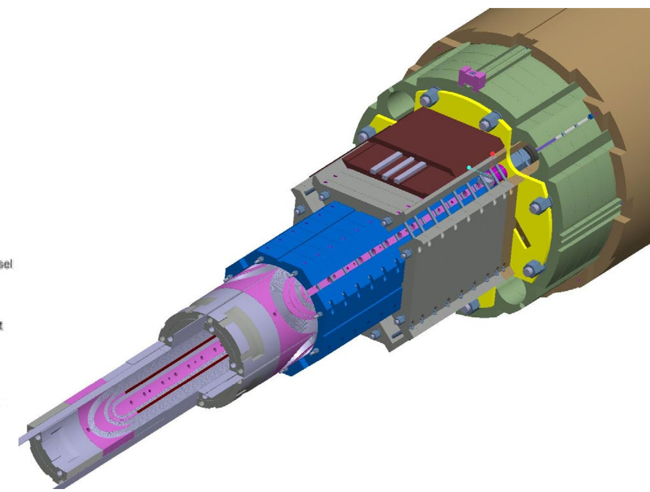
Superconducting Magnets Cont.





Tevatron NbTi superconducting dipole magnet





LHC Nb_3Sn superconducting Quadrupole magnet

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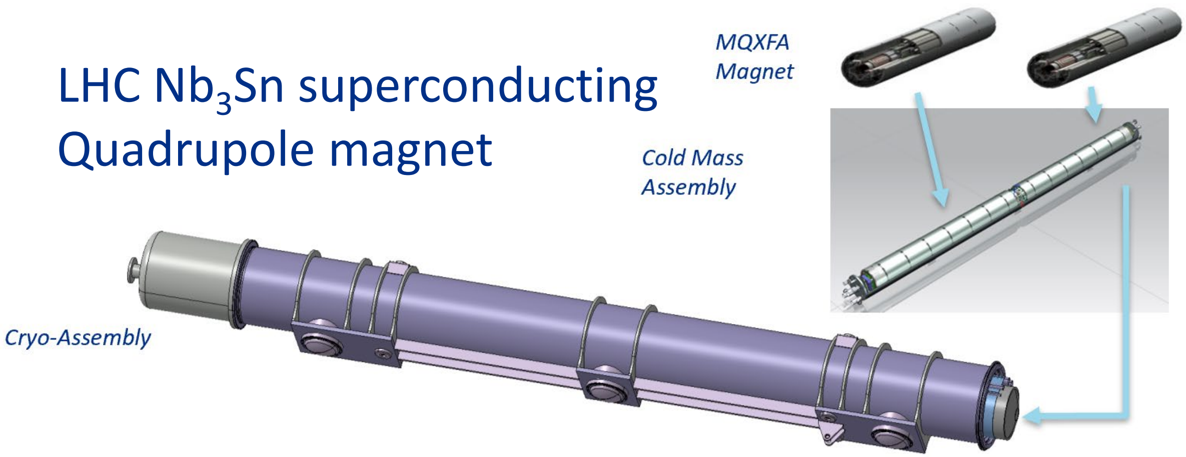
Superconducting Magnets Cont.

- Comparing with the conventional magnets, the superconducting magnets are able to
  - Produce large volumes of high field for a small consumption of electric power.
  - Operate at high overall current density allowing a given number of ampere-turns to be provided by a rather small volume of winding.
  - Produce higher gradients of field.
- To Maintain the low-temperature environment, superconducting magnets must be placed in special liquid helium vessels or cryostats, which are vacuum-insulated containers.
- The only power required by a superconducting magnet is the refrigeration power needed to cool it to low temperature and a small current supply needed to initiate the flow of current round the circuit.
- The fabrication of the superconducting magnet costs $$$. Keep them cheap…

LHC Nb_3Sn superconducting Quadrupole magnet

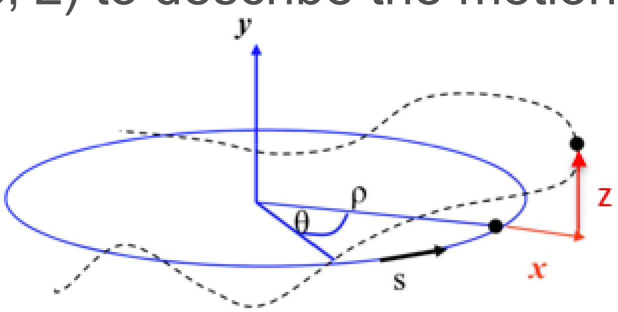


Particle Beam in Accelerator

- When an accelerator is constructed, the nominal trajectory of the particle beam is fixed.
- This trajectory may simply be a straight line, as is the case in linear accelerator.
- In circular machines such as storage rings, however, it may have a very complicated shape consisting of numerous curves connected by straight sections of various lengths.
- The trajectories of individual particles within a beam always have a certain angular divergence, and without further measures they would eventually hit the wall of the vacuum chamber and be lost.
- Therefore, after the particle trajectory is fixed, steer the diverging particles back onto the ideal trajectory by means of electromagnetic fields (E and B).

Magnets to Steer the Beams

- Particles of charge e and velocity v experience the Lorentz force $F = q(E + v \times B)$.
- At relativistic velocities electric fields E and magnetic fields B have the same effect if $E = cB$. For B = 1 T = 1 Vs/m^2 and c = 3x10^8 m/s ➔ E = 3x10^8 V/m = 300 MV/m
- The maximum achievable electric gradients with RF cavities technologies are ~ 100 MV/m
- To steer the particle beams, we almost always use magnets.
- Below is the coordinate system (x, s, z) to describe the motions of particles along the trajectory.



- For simplicity we assume:
  1. Particles only move along the s direction: v = (0, v_s, 0)
  2. The magnetic field only has transverse components: B = (B_x,0,B_z)
  3. The bending radius for the particles in the accelerator: ρ

Beam Rigidity Formula

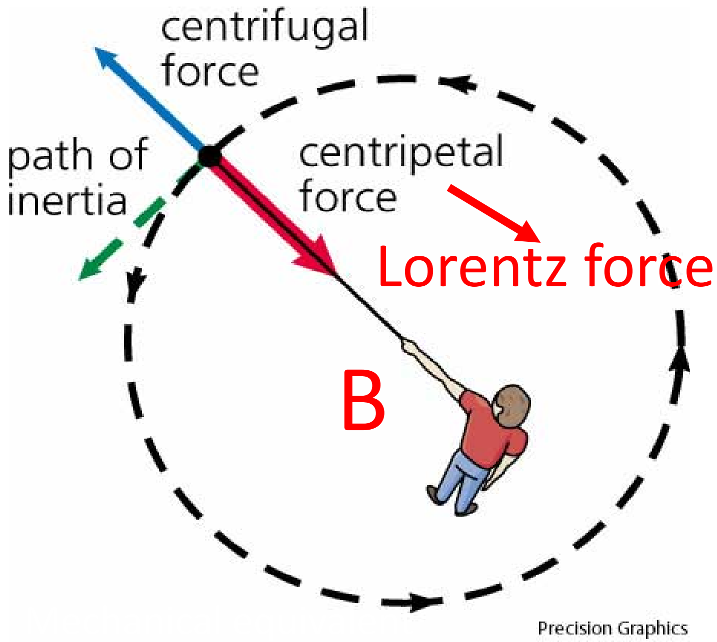
- For a particle moving in the horizontal plane through the magnetic field there is a balance between the Lorentz force and the centrifugal force

$$q \cdot v_s \cdot B_z = \frac{m \cdot v_s^2}{\rho}$$

$$p = m \cdot v$$

$$B_z \rho = \frac{p}{q}$$

Beam rigidity formula



The maximum field of the dipoles limit the beam energy, though magnetic field does not accelerate the beam at all.

Magnetic Field

- Since the beam dimensions and beam displacement around the nominal trajectory are usually much smaller than the curvature radius (e.g. LHC beam size ~ 1 mm to few μm, and the beam excursions around the nominal orbit are of ~ 2 mm, while LHC curvature radius ~ 2.8 km) we can expand the magnetic field in the vicinity of the nominal trajectory:

$$B_z(x) = B_{z0} + \frac{dB_z}{dx}x + \frac{1}{2!}\frac{d^2B_z}{dx^2}x^2 + \frac{1}{3!}\frac{d^3B_z}{dx^3}x^3 + \cdots$$

- Multiply by q/p

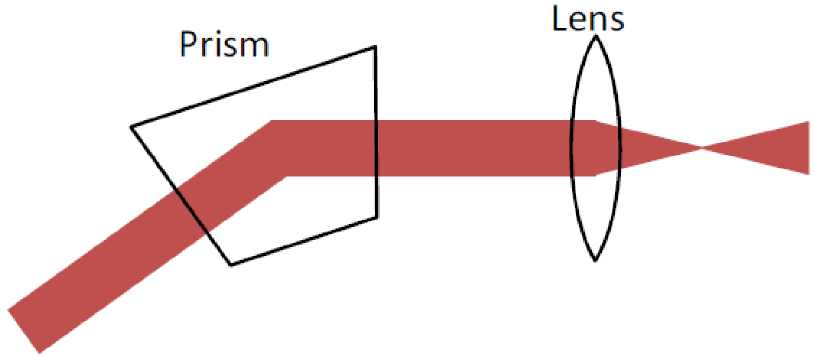
$$\frac{q}{p}B_z(x) = \frac{q}{p}B_{z0} + \frac{q}{p}\frac{dB_z}{dx}x + \frac{1}{2!}\frac{q}{p}\frac{d^2B_z}{dx^2}x^2 + \frac{1}{3!}\frac{q}{p}\frac{d^3B_z}{dx^3}x^3 + \cdots$$

DIPOLE QUADRUPOLE SEXTUPOLE OCTUPOLE

$$= \frac{1}{\rho} + kx + \frac{1}{2!}mx^2 + \frac{1}{3!}ox^3 + \cdots$$

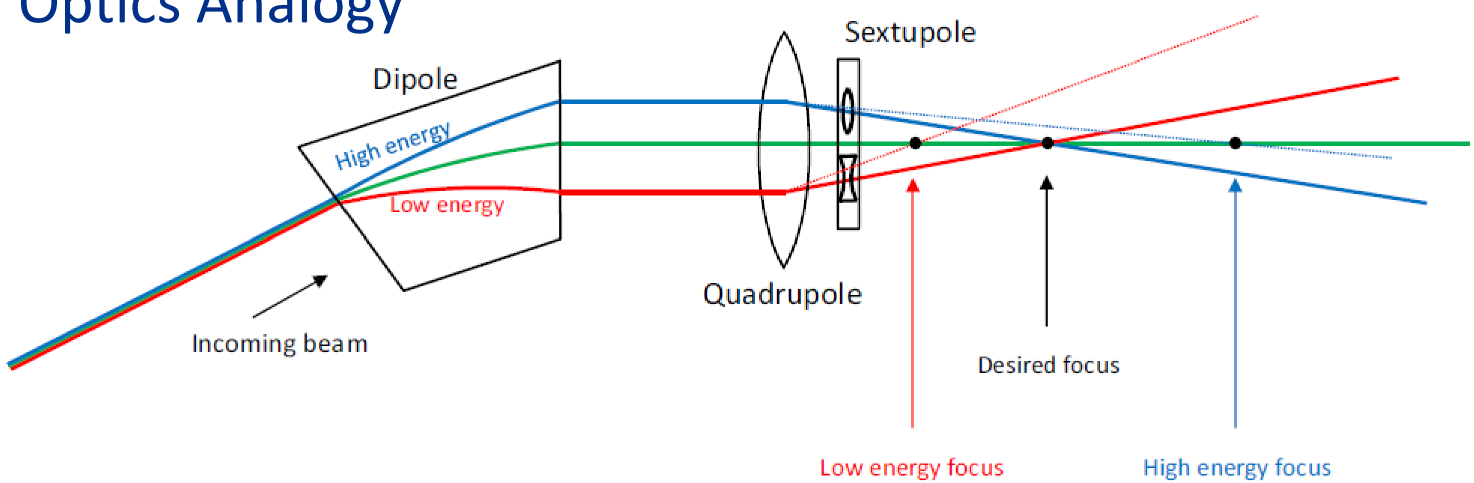
- The magnetic field around the beam is a sum of multipoles; each one has a different effect on the particles.

Magnet Types in Accelerator



multipole	definition	effect
dipole	$\frac{1}{R} = \frac{e}{p} B_{z0}$	beam steering
quadrupole	$k = \frac{e}{p} \frac{dB_z}{dx}$	beam focusing
sextupole	$m = \frac{e}{p} \frac{d^2 B_z}{dx^2}$	chromaticity compensation
octupole	$o = \frac{e}{p} \frac{d^3 B_z}{dx^3}$	field errors or field compensation
etc.	.	
Solenoid		beam focusing

Optics Analogy



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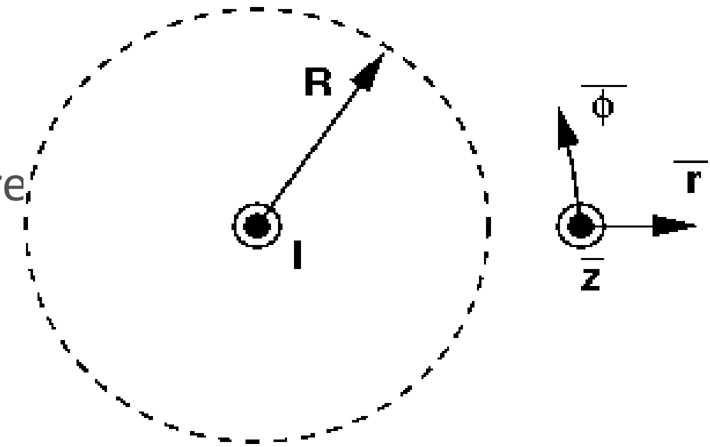
Chapter 3 Magnetic Field Equations

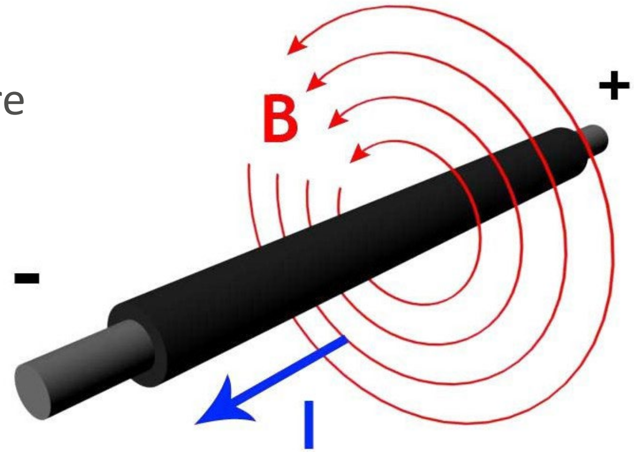
Magnetic Field Around a Very Long Wire Carrying Current

- Within a short time after Oersted's discovery, Biot Savart experimentally formulated an equation to determine the magnetic flux density $\vec{B}$ (Tesla or Newton per meter per Ampere or Weber per square meter) at a point produced by a current-carrying conductor.
- If the conductor is infinitely long



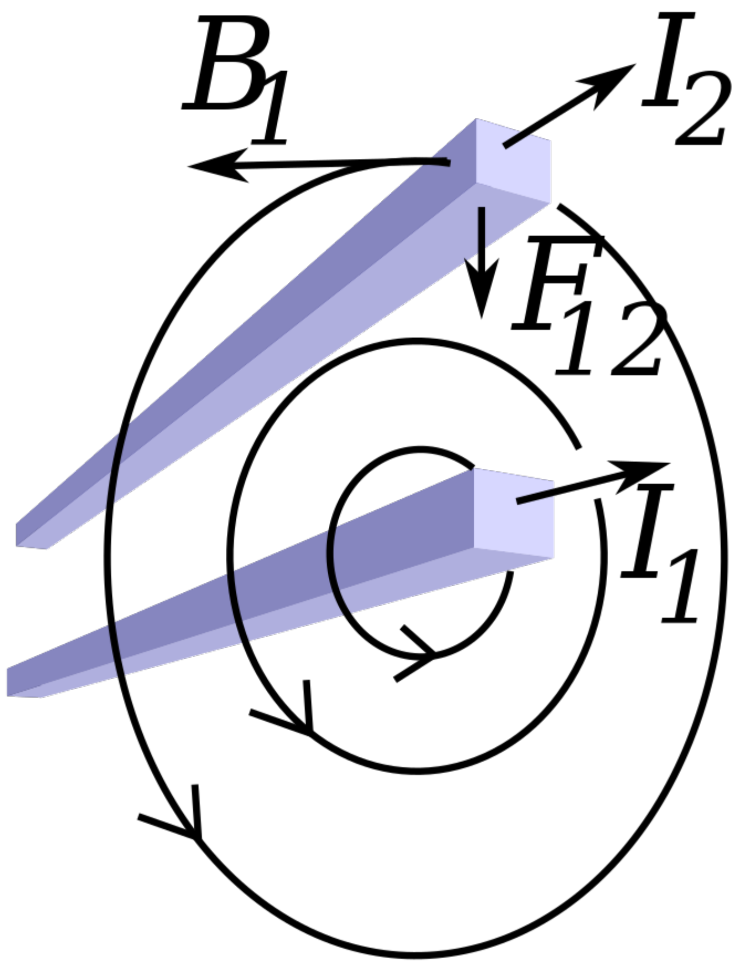
$$\vec{B} = \frac{\mu_0 I}{2\pi R}\,\vec{\phi}, \qquad \vec{H} = \vec{B}/\mu_0$$

H - magnetic field intensity (A/m) or magnetizing pressure



- ❖ The magnetic field is rotationally symmetric around the wire.
- ❖ The magnetic field is proportional to the current in the wire and 1/R.

Ampere's Force Law



- When put conductor #2 (I_2) by conductor #1 (I_1), and the magnetic force exerted by conductor #1 upon conductor #2.

$$\overrightarrow{F_{12}} = \overrightarrow{I_2} \cdot L \times \overrightarrow{B_1}$$

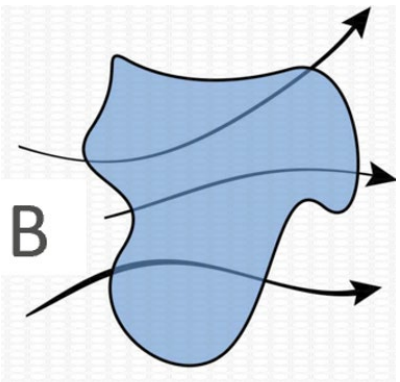
❖ Example: A coil of 3 m length with I = 50 A immerses in a perpendicular magnetic field of B = 1.5 T.

F = 50 (A) ·3 (m) ·1.5 (T) =225 (N)

Gauss's Law and Ampere's Law for Magnetic Field

Gauss's Law

$$\oint_s \vec{B} \cdot \overrightarrow{ds} = 0 \; or \; \nabla \cdot \vec{B} = 0$$

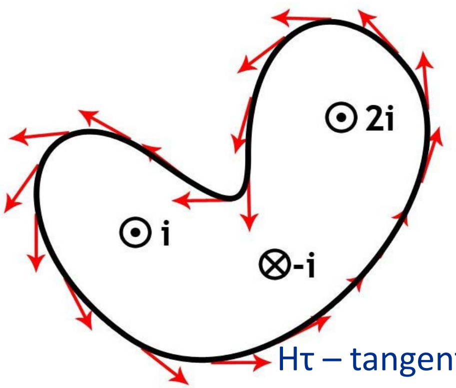


- No magnetic monopoles
- No net magnetic flux enters or exits a closed surface.
- What goes in must come out.
- Magnetic flux lines never terminate. They are closed on themselves in loops.

Ampere's Law

$$\int_C \overline{H} \cdot d\bar{l} = \int_S \overline{J} \cdot d\overline{A} = I_{enclosed}$$

$$or \; \nabla \times \vec{H} = \vec{j}$$



- The line integral of the magnetic field intensity around a closed path equals the current enclosed.

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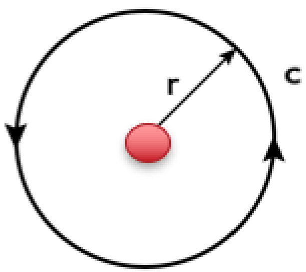
Ampere's Law Examples

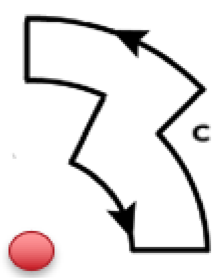
Current I

(a) Circular path enclosing wire



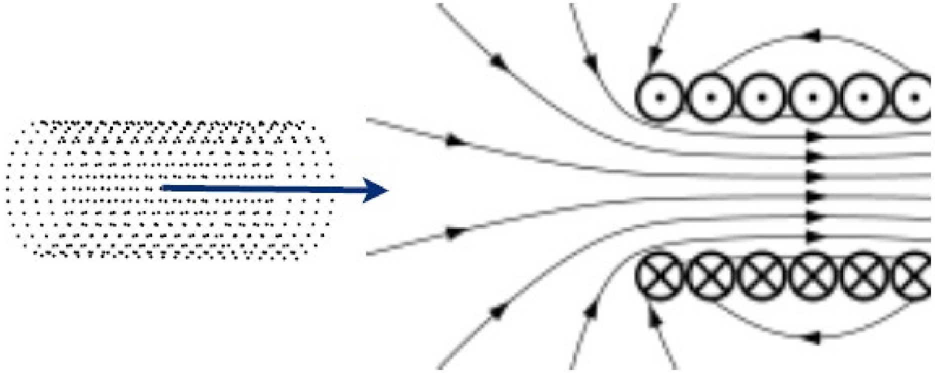
$$\oint_C \vec{H} \cdot \vec{dl} = \int_0^{2\pi} H_\phi r d\phi = 2\pi r H_\phi = I$$

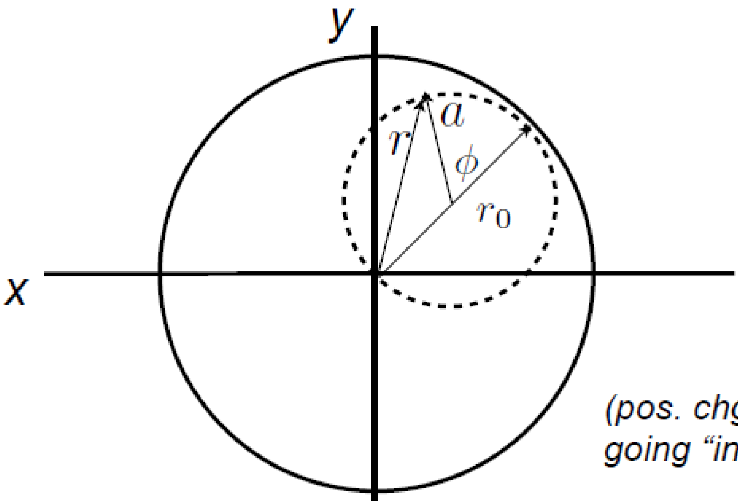
(b) Path lying in plane perpendiaculr to wire



$$\oint_C \vec{H} \cdot \vec{dl} = 0$$

Solenoid Field

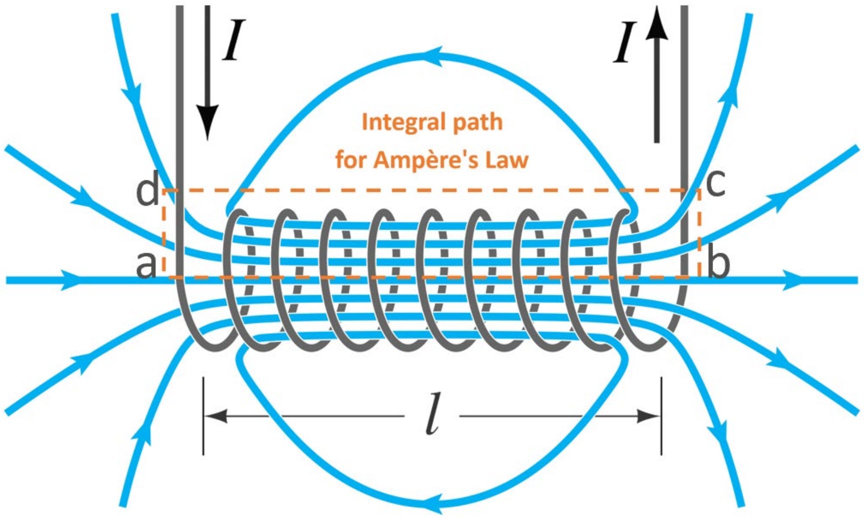


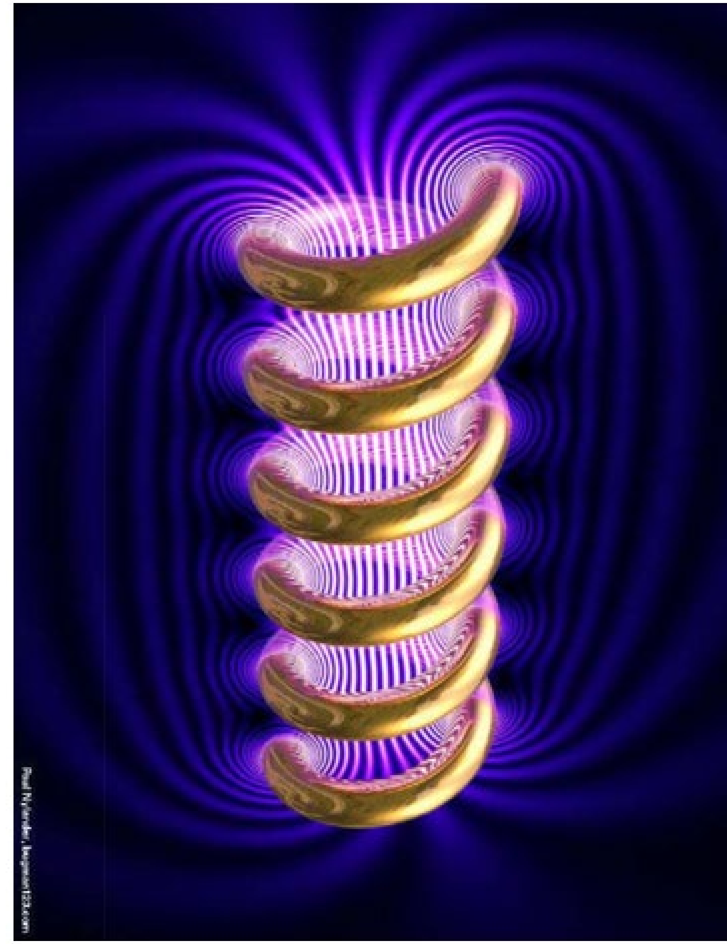


(pos. chg. particles going "into" the page)

- Particles travel in the same direction of the B field, how does a solenoid "focus"?
- When the beam enters the solenoid, it gains angular momentum, resulting in helical trajectory by an amount of ϕ.
- Upon exit, assuming the rotation angle is small, the angular momentum of the beam will be removed.
- Solenoids often used with lower-energy beams.

Ampere's Law on a Solenoid



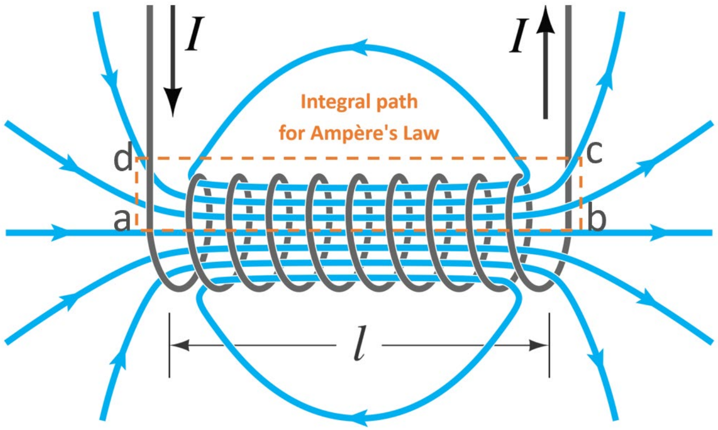


$$\oint_C \vec{H} \cdot \vec{dl} = \int_a^b H_{inside}\, dl + \int_b^c \cancelto{0}{H_{end}}\, dl + \int_c^d \cancelto{0}{H_{outside}}\, dl + \int_d^a \cancelto{0}{H_{end}}\, dl = H_{inside} \cdot l = NI$$

$$H_{inside} \cong \frac{NI}{l}$$

$$B_{inside} = \mu_0 \cdot H_{inside}$$

Example: Solenoid



$$H_{inside} \cong \frac{NI}{l}$$

$$B_{inside} = \mu_0 \cdot H_{inside} = \frac{\mu_0 NI}{l}$$

$$\mu_0 = 4\pi \times 10^{-7}\ (T \cdot m/A)$$

- *Given*
- *N = 80 turns*
- *I = 13 A*
- *B = 7.3 x 10−3*

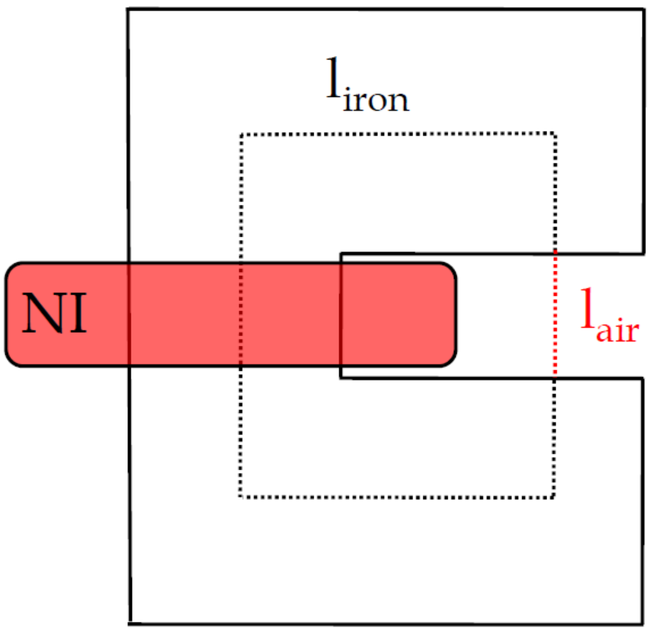
A solenoid formed from a length of wire has 80 turns, carrying a constant current of 13 A and the strength of the magnetic field produced is measured to be 7.3 ×10^{-3} *T at its center. Calculate the length of the solenoid.*

Solution

- $l = \frac{\mu_0 NI}{B} = \frac{4\pi \times 10{-}7 \times 80 \times 13}{7.3 \times 10^{-3}} = 0.179$ m

Thus, the length of the solenoid is 0.179 m

Ampere's Law on a Dipole



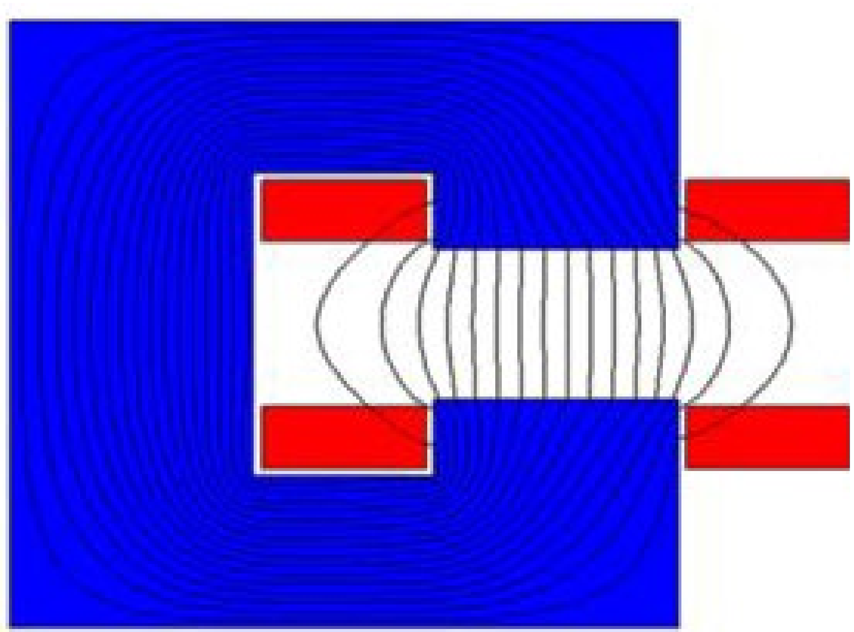
Material	$\mu(Hm^{-1})$	μ_r
Vacuum	4π E-07 (μ_0)	1
Air	1.25663753 E-06	1.00000037
Copper	1.256629 E-06	0.999994
Aluminum	1.256665 E-06	1.000022
Carbon steel	1.26 E-04	100
Iron (99.8% pure)	6.3 E-03	5000
Permalloy	1.25 E-01	100000

$$\oint_C \vec{H} \cdot \vec{dl} = H_{iron} \cdot l_{iron} + H_{air} \cdot l_{air} = NI$$

$$\frac{B_{iron}}{\mu_0 \cdot \mu_r} \cdot l_{iron} + \frac{B_{air}}{\mu_0} \cdot l_{air} = NI$$

- μ_r for the iron is 5000
- $\frac{B_{iron}}{\mu_0 \cdot \mu_r} \cdot l_{iron} \ll \frac{B_{air}}{\mu_0} \cdot l_{air}$
- $NI \cong \frac{B_{air}}{\mu_0} \cdot l_{air}$

Example: Dipole



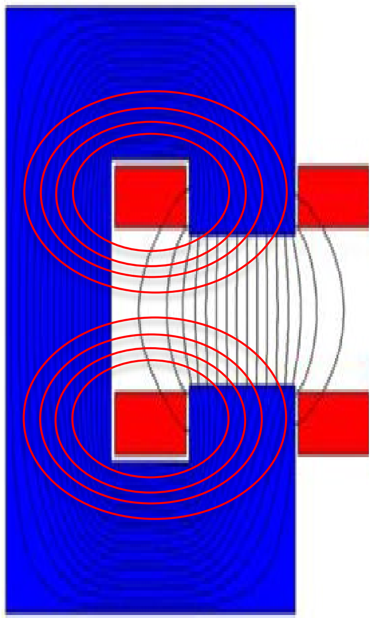
A C-shape dipole is needed to provide 1 T magnetic flux density at the center of the 50 mm air gap. The maximum current 500 A can be provided by a power supply. Calculate the number of turns requires for the coil.

- $NI \cong \frac{B_{air}}{\mu_0} \cdot l_{air}$

- $\mu_0 = 4\,\pi \times 10^{-7}\ (T \cdot m/A)$
- $I = 500\ A$
- $B_{air} = 1\ T$
- $l_{air} = 0.05$ m

- $N = \frac{B_{air}}{\mu_0 I} \cdot l_{air} = 80\ turns$

Example: Dipole



A C-shape dipole is needed to provide 1 T magnetic flux density at the center of the 50 mm air gap. The maximum current 500 A can be provided by a power supply. Calculate the number of turns requires for the coil.

- $N = 80\ turns$

- $\mathrm{B} < 1T$

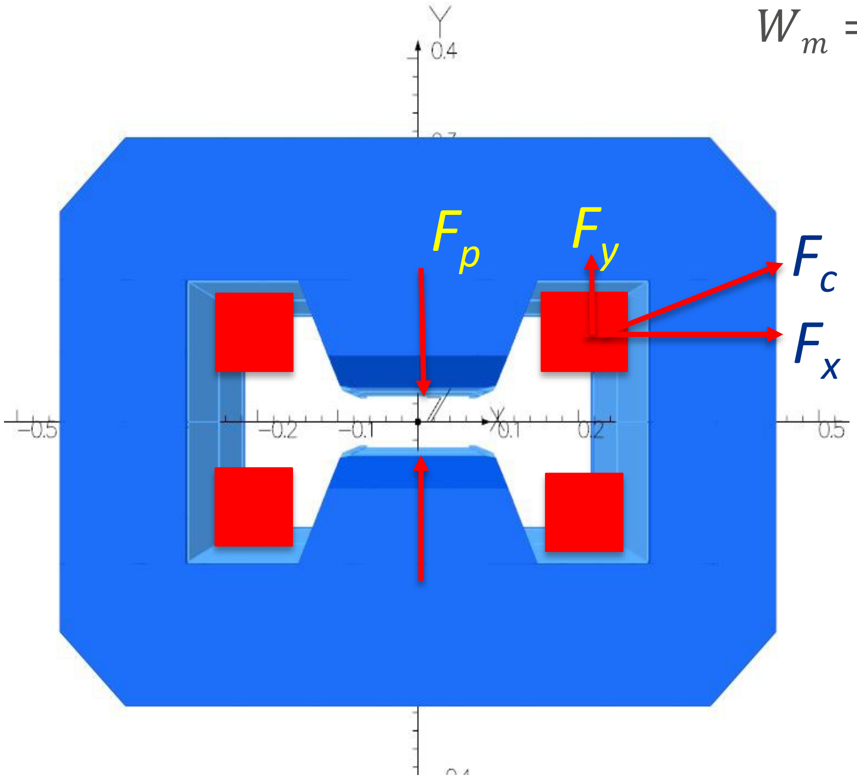
Energy and Forces

- In magnetostatic, the energy density in a magnetic field is

$$w_m = \frac{1}{2}\vec{B}\cdot\vec{H} = \frac{1}{2}B\cdot H = \frac{1}{2\mu}B^2$$

- The total energy in a volume is

$$W_m = \int_v w_m \, dv$$



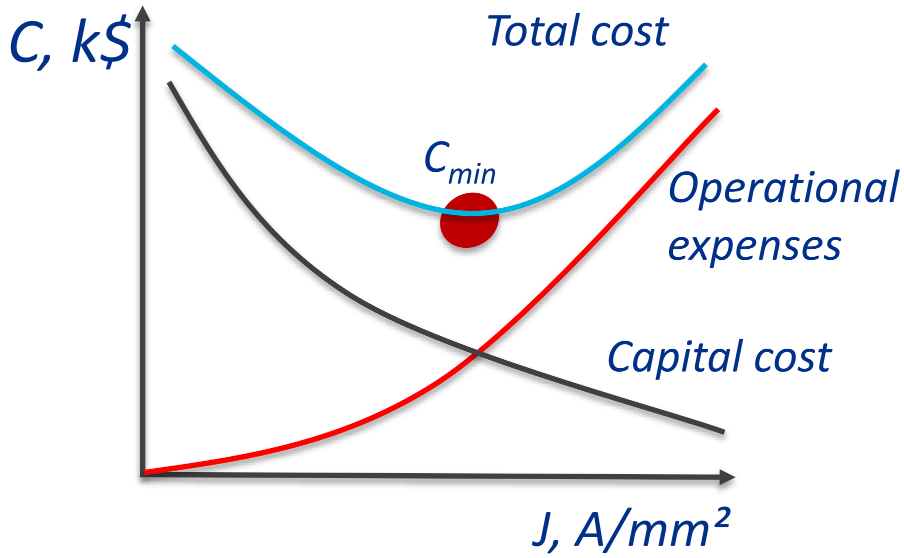
Example:

- *In a dipole magnet, more than 90% of magnet stored energy concentrated in the magnet air gap because* ***H*** *in the iron yoke is very small.*
- *Work (energy) done by a force, and the force between the poles is* $F = \frac{dW_m}{dl_{air}} = \frac{1}{2\mu_0}B^2A$

Magnet Resistance and Inductance

- *Magnet coil resistance:* $R_T = R_0(1 + \alpha\Delta T), R_0 = \frac{\rho L}{A}$
  - R_0: *Resistance at room temperature [Ω]*
  - R_T: *Resistance at the temperature of T [Ω]*
  - α : *Temperature coefficient, and for Cu, α = 0.004 [1/C°]*
  - ΔT: *Temperature difference between T and room temperature [C°]*
  - *L: Conductor length [m]*
  - *A: Conductor cross-section area [m^2]*
- *Magnet coil inductance:* $L = \frac{2W_m}{I^2}$
  - L: *Inductance [H/m]*
  - W_m: *Stored energy [J]*
  - I: *Current [A]*

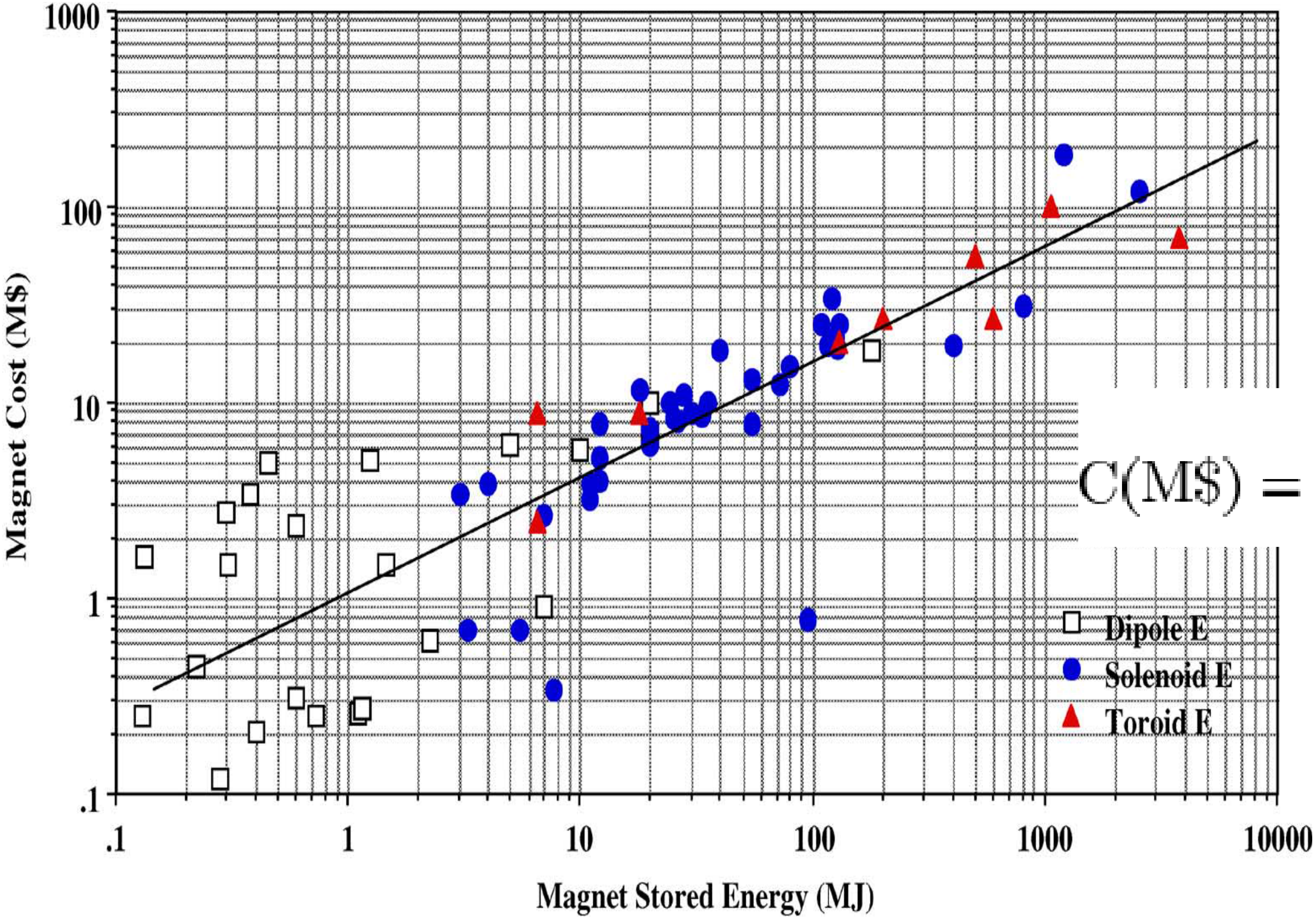
Magnet Cost



- ✓ *Cost of magnets:*
- \- *Small – 10k$- 50k$;*
- \- *Medium – 50k$ - 100k$;*
- \- *Large - > 100 k$.*
- \- *Cost of prototype is at least 3 times more than at a serial production.*

- ✓ *Capital cost includes cost of conductors, materials, tooling, and fabrication.*
- ✓ *Operational expenses include cost of electricity, water or LHe cooling for 10 years of operation at 5000-7000 hours/year.*
- ✓ *Optimal current density for air cooled magnets is 1.5 A/mm² - 2.0 A/mm².*
- ✓ *Optimal current density for water cooled magnets is 4.0 A/mm².*
- ✓ *For superconducting magnets, the operational peak current density should be below critical, at least 20%, measured for the short sample.*

Magnet Stored Energy vs. Cost



$$C(M\$) = 0.92\,[E(MJ)]^{0.60}$$

- *The Cost of Superconducting Magnets as a Function of Stored Energy and Design Magnetic Induction Times the Field Volume. (Michael A. Green and Bruce P. Strauss)*

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Appendix

Ref. Book: The Physics of Particle Accelerators, Klaus Wille

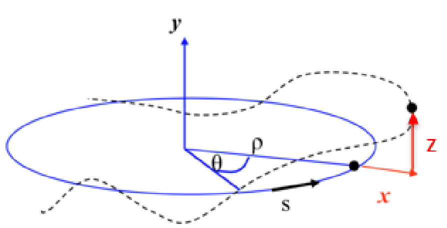
Design of Iron Pole Magnets

How magnets must be shaped to produce a dipole field, quadrupole field and so on?

- Ampere's Law $\nabla \times \vec{H} = \vec{j}$. In the accelerator, the field shape is important only in the vicinity of the beam, where no current is flowing, so $\nabla \times \vec{H} = 0 \; or \; \nabla \times \vec{B} = 0 \rightarrow \vec{B} = \nabla\Phi$ (Φ is a scale potential)
- Gauss's Law $\nabla \cdot \vec{B} = 0$, so $\nabla \cdot \nabla\Phi = \nabla^2\Phi = 0$
- $\vec{B} = \nabla\Phi$ **and** $\nabla^2\Phi = 0$ **form the theoretical basis of the design of iron pole magnets.**
- In the particle beam coordinate system, $v = (0, v_s, 0)$ and $B = (B_x, 0, B_z)$. Now define $B_z(x,z) = G_z(x) + f(z)$, where $B_z(x,z)$ is z component of the magnetic field, $G_z(x)$ is the z component depend only on the x-axis, and $f(z)$ is an unknow function for the component depends only on z direction.
  - $\vec{B} = \nabla\Phi \rightarrow B_z = \frac{d\Phi(x,z)}{dz} \rightarrow \Phi(x,z) = \int B_z dz = G_z(x)z + \int f(z)\, dz$
  - $\nabla^2\Phi = \frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial z^2} = 0 \rightarrow \frac{d^2G_z(x)}{dx^2}z + \frac{df(z)}{dz} = 0 \rightarrow f(z) = -\int \frac{d^2G_{z(x)}}{dx^2} z dz = -\frac{1}{2}\frac{d^2G_{z(x)}}{dx^2}z^2$

$$\Phi(x,z) = G_z(x)z - \frac{1}{6}\frac{d^2G_{z(x)}}{dx^2}z^3$$

$$B_x(x,z) = \frac{d\Phi(x,z)}{dx}, B_z(x,z) = \frac{d\Phi(x,z)}{dz}$$



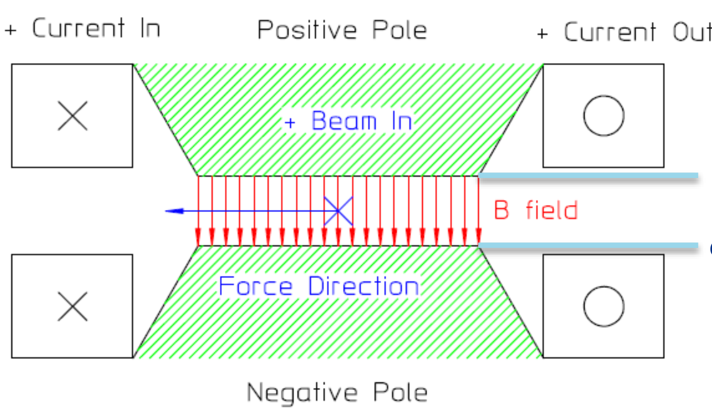
The potential and the magnetic field in the entire x-z plane can be calculated for any field shape $G_z(x)$ along the x-axis.

Dipole Magnetic Field

$$\frac{q}{p}B_z(x) = \frac{q}{p}B_{z0} + \frac{q}{p}\frac{dB_z}{dx}x + \frac{1}{2!}\frac{q}{p}\frac{d^2B_z}{dx^2}x^2 + \frac{1}{3!}\frac{q}{p}\frac{d^3B_z}{dx^3}x^3 + \cdots$$
$$= \frac{1}{\rho} + kx + \frac{1}{2!}mx^2 + \frac{1}{3!}ox^3 + \cdots$$

DIPOLE

$$\Phi(x,z) = G_z(x)z - \frac{1}{6}\frac{d^2G_z(x)}{dx^2}z^3$$
$$B_x(x,z) = \frac{d\Phi(x,z)}{dx}, B_z(x,z) = \frac{d\Phi(x,z)}{dz}$$



$\Phi = B_0 g/2$

$\Phi = -B_0 g/2$

- To bend charged particles around a circular path, dipole magnets are used, which has a constant field $\frac{1}{\rho}$ along the x-axis
- Field shape $G_z(x) = B_0, \frac{d^2G_z(x)}{dx^2} = 0$
- $\Phi(x,z) = B_0 z$
- $B_x(x,z) = 0, B_z(x,z) = B_0$
- The equipotential lines are parallel to the x-axis $z = \frac{\Phi}{B_0}$.
- A dipole consists of two parallel iron poles of separation air gap g.

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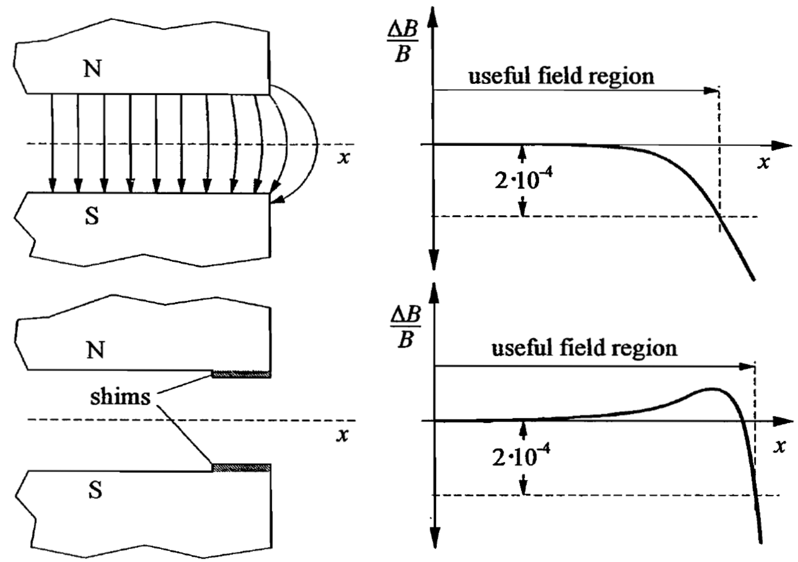
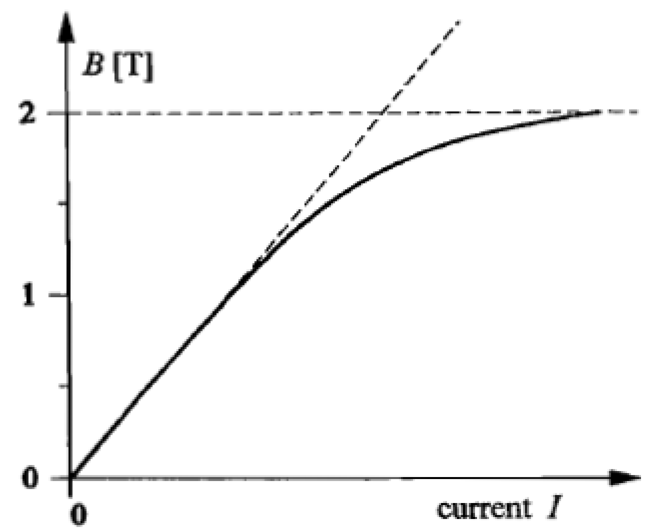
Dipole Magnet

In a realistic magnet, there will be deviations from this ideal field.

1. Homogeneous field could only be produced by infinitely long poles. At a certain horizontal distance from the center of the magnet, the field falls away as the field lines at the edge of the poles are pushed outwards. To compensate, use shims to the ends of the poles.
2. Another problem is that iron saturates at ~1 T, and above 1 T the field lags behind the current and levels off at a constant value of ~2 T. It is pointless to increase the current any further.



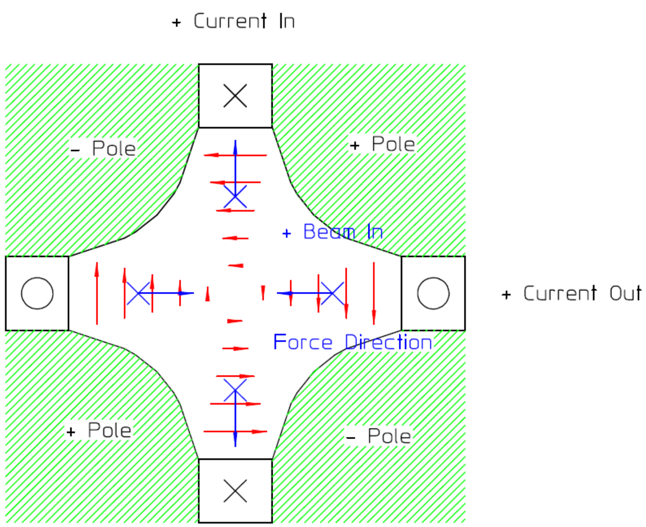


Quadrupole Magnetic Field

$$\frac{q}{p}B_z(x) = \frac{q}{p}B_{z0} + \frac{q}{p}\frac{dB_z}{dx}x + \frac{1}{2!}\frac{q}{p}\frac{d^2B_z}{dx^2}x^2 + \frac{1}{3!}\frac{q}{p}\frac{d^3B_z}{dx^3}x^3 + \cdots$$
$$= \frac{1}{\rho} + kx + \frac{1}{2!}mx^2 + \frac{1}{3!}ox^3 + \cdots$$

QUADRUPOLE

$$\Phi(x,z) = G_z(x)z - \frac{1}{6}\frac{d^2G_z(x)}{dx^2}z^3$$
$$B_x(x,z) = \frac{d\Phi(x,z)}{dx}, B_z(x,z) = \frac{d\Phi(x,z)}{dz}$$



- To focus the beams, quadrupole fields are used, which according to the equation above (kx), disappear along the beam axis and increase linearly along x-axis
- Field shape $G_z(x) = gx \; with \; g = \frac{dB_z}{dx}$
- $\frac{d^2G_{z(x)}}{dx^2} = 0, so \; \Phi(x,z) = gxz$
- $B_x(x,z) = gz, B_z(x,z) = gx$
- The equipotential lines are hyperbolae of $z = \frac{\Phi}{gx}$
- A quadrupole consists of four iron poles with hyperbolic surfaces, arranged with alternating polarity. And the four poles are excited by coils which surround them.

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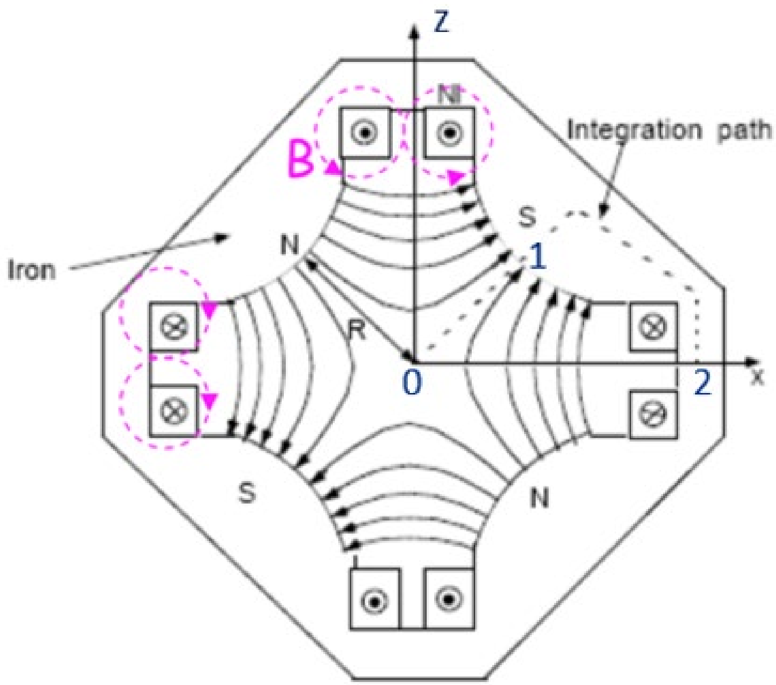
Ampere's Law on a Quadrupole



$$\oint_C \vec{H}\cdot\vec{dl} = \int_0^1 H\,dl + \int_1^2 \overset{\sim 0}{H_{iron}}\,dl + \int_2^0 \overset{0}{H}\,dl = NI$$

$$NI \cong \int_0^1 H\,dl$$

The field between the beam axis and the pole:

$B_x(x,z) = gz, B_z(x,z) = gx$

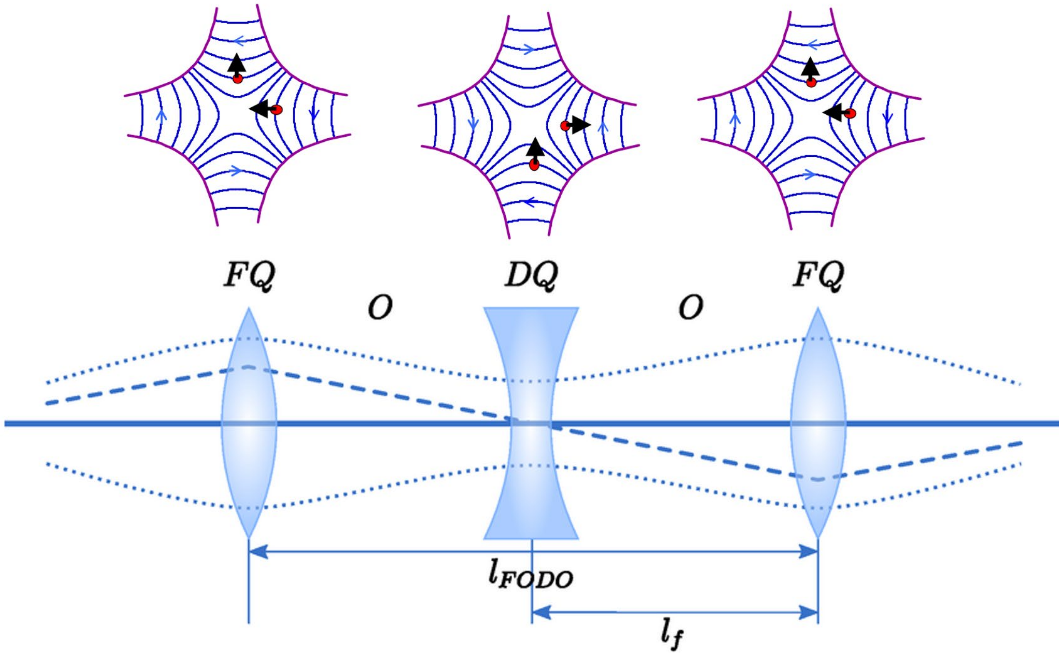
Along the line 0-1, $B(x,z) = g\sqrt{x^2+z^2} = gr$

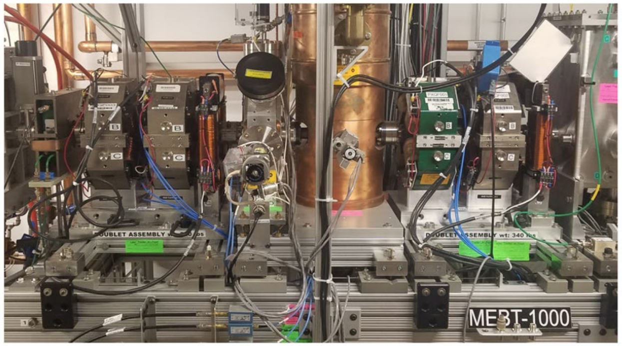
$NI = \int_0^R H\,dr = \int_0^R \frac{B(x,z)}{\mu_0} dr = \int_0^R \frac{g}{\mu_0} r\,dr = \frac{g}{\mu_0}\frac{R^2}{2})$

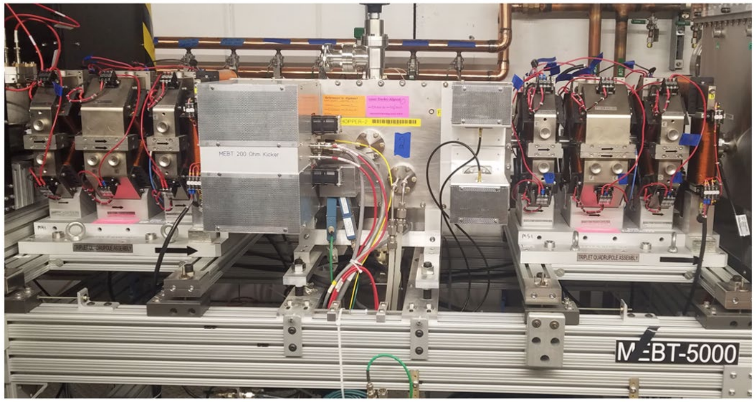
$$\boldsymbol{g = \frac{2\mu_0 NI}{R^2}}$$

- Keep the pole separation R as small as possible for high-strength quadrupoles, so that the current I and hence the power consumption of the magnet are reduced.
- Similar to the dipole magnet, the perfect quadrupole field could only be produced by infinitely hyperbolic pole surfaces. In a quadrupole magnet, shims may be needed for compensation.

Quadrupole Magnets







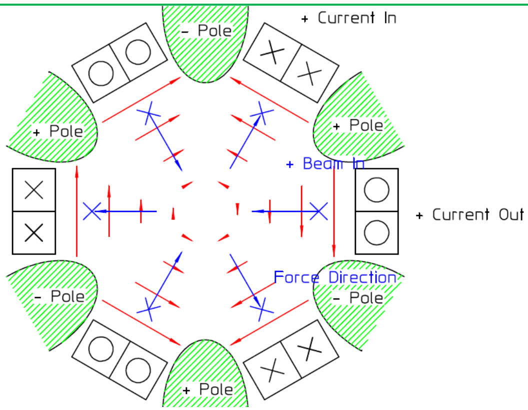
- The distribution of magnetic field lines between the poles causes a quadrupole magnet which focuses in the horizontal plane to defocus the beam in the vertical direction.
- To properly focus the beam, it is therefore necessary to use at least two quadrupoles, rotated through 90° relative to each other.

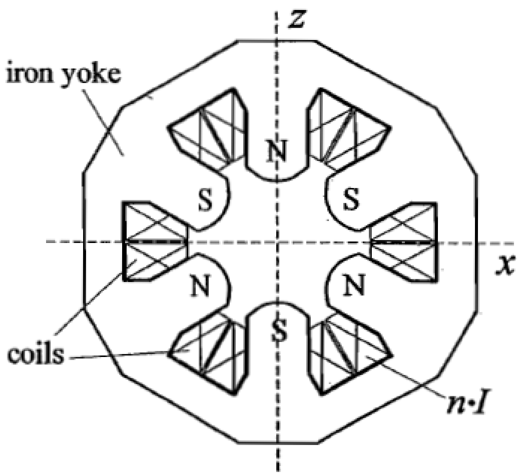
Sextupole Magnetic Field

$$\frac{q}{p}B_z(x) = \frac{q}{p}B_{z0} + \frac{q}{p}\frac{dB_z}{dx}x + \frac{1}{2!}\frac{q}{p}\frac{d^2B_z}{dx^2}x^2 + \frac{1}{3!}\frac{q}{p}\frac{d^3B_z}{dx^3}x^3 + \cdots$$
$$= \frac{1}{\rho} + kx + \frac{1}{2!}mx^2 + \frac{1}{3!}ox^3 + \cdots$$

SEXTUPOLE

$$\Phi(x,z) = G_z(x)z - \frac{1}{6}\frac{d^2G_z(x)}{dx^2}z^3$$
$$B_x(x,z) = \frac{d\Phi(x,z)}{dx}, B_z(x,z) = \frac{d\Phi(x,z)}{dz}$$





- Sextupole magnet is primarily used to compensate for chromatic abberation in strongly focusing magnetic structures.
- Field shape along x-aixs $G_z(x) = \frac{1}{2}g'x^2, g' = \frac{d^2G_{z(x)}}{dx^2}$
- $\Phi(x,z) = \frac{1}{2}g'(x^2z - \frac{z^3}{3})$. In the sextupole where $f(z) \neq 0$, the particle motion in the horizontal plane is coupled to that in the vertical plane, and vice-versa.
- $B_x(x,z) = g'xz, \ B_z(x,z) = \frac{1}{2}g'(x^2 - z^2)$
- The equipotential lines are $x = \sqrt{\frac{2\Phi}{g'z} + \frac{z^2}{3}}$, which determines the shape of the six poles, arranged with alternating polarity, each at an angle of 60° to the next.
- $g' = \frac{d^2G_{z(x)}}{dx^2} = \frac{6\mu_0 NI}{R^3}$